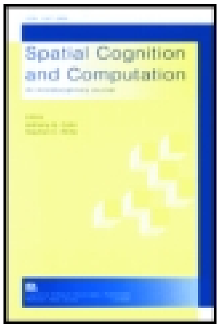

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Modelling Dynamic Spatial Systems in the Situation Calculus

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Abstract: We propose and systematically formalise a dynamical spatial systems approach for the modelling of changing spatial environments. The formalisation adheres to the semantics of the situation calculus and includes a systematic account of key aspects that are necessary to realize a domain-independent qualitative spatial theory that may be utilised across diverse application domains. The spatial theory is primarily derivable from the all-pervasive generic notion of "qualitative spatial calculi" that are representative of differing aspects of space. In addition, the theory also includes aspects, both ontological and phenomenal in nature, that are considered inherent in dynamic spatial systems. Foundational to the formalisation is a causal theory that adheres to the representational and computational semantics of the situation calculus. This foundational theory provides the necessary (general) mechanism required to represent and reason about changing spatial environments and also includes an account of the key fundamental epistemological issues concerning the frame and the ramification problems that arise whilst modelling change within such domains. The main advantage of the proposed approach is that based on the structure and semantics of the proposed framework, fundamental reasoning tasks such as projection and explanation directly follow. Within the specialised spatial reasoning domain, these translate to spatial planning/re-configuration, causal explanation and spatial simulation. Our approach is based on the hypothesis that alternate formalisations of existing qualitative spatial calculi using high-level tools such as the situation calculus are essential for their utilisation in diverse application domains such as intelligent systems, cognitive robotics and event-based GIS.

Keywords: dynamic spatial systems, qualitative spatial reasoning, reasoning about actions and change

1. MOTIVATION

Most research in *Qualitative Spatial Reasoning* (QSR) has focussed on the construction of formal methods for spatial abstraction and reasoning. This

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has primarily resulted in the development of qualitative spatial calculi that are representative of distinct spatial domains—topology, orientation, distance, direction, size, and shape (Cohn and Hazarika, 2001a present a comprehensive review). However, alternate formalisations of existing spatial calculi or their integration within general logic-based commonsense reasoning frameworks, which is essential for their applicability in realistic domains, has not been given adequate attention. Relatively little work has explicitly addressed the need for modelling spatial calculi using alternate formalisations that could be directly applied toward the modelling of dynamically varying spatial systems, e.g., in the form of spatial control or planning in cognitive robotics, spatial decision-support in intelligent systems and explanatory models in event-based geographic information systems. The integration of specialised spatial representation and reasoning techniques within general commonsense reasoning frameworks is an important next-step for their applicability in realistic domains. This integration is nontrivial and requires unification along ontological, representational and computational fronts. Indeed, it is also closely related to the much general problem pertaining to the subdivision of endeavours in *Artificial Intelligence* (AI) and the development of a unifying semantics for logic-based common-sense reasoning and other specialised reasoning domains such as qualitative spatial reasoning (McCarthy, 1977). Broadly, it is this '*integration*' aspect that has motivated the research described in this article.

A Causal Perspective to Spatial Reasoning. Viewed through the general prism of a dynamic system (Sandewall, 1994), reasoning within the class of application domains involving the representation of dynamically changing spatial systems (e.g., cognitive robotics) includes representation primarily along two fronts: (a) Modelling of the underlying *qualitative physics* of the spatial domain, which, depending on the richness of the spatial theory under consideration, could be modelled as changing relationships relevant to differing aspects of space such as topology, orientation and size. (b) Elaborations involving the representation of the *causal* (i.e., a cause and effect based characterisation of the system) and possibly the *teleological* or purpose-directed aspects of spatial change in situations where purpose/goal is applicable. Whereas the underlying qualitative physics (i.e., qualitative spatial reasoning) is concerned with the manner in which a set of spatial relationships evolve during a certain time interval, reasoning about the causal and teleological aspects of spatial change encompasses reasoning about events, actions and their effects. For example, consider a simulated environment consisting of an agent that needs to travel from location L_1 to location L_2 via a sequence of spatial transformations that are *affected* by certain spatial control actions (e.g., *turn-left*, *turn-around*). Minimally, there are two main closely related aspects to this problem:

(1) Spatial: The specific sequence of spatial transformations needed in order to achieve a certain desired spatial configuration as well as its *legality* or *consistency* with regard to a set of spatial dynamics.

(2) Causal: A causal characterisation, in terms of spatial causes and effects, of the agent's spatial control actions (or of other occurrences within system), which effectuate the required transformation of the underlying spatial structures being modelled, and the overall goal or the telic aspect of achieving a desired spatial configuration, which dictates the reason the agent wants to move from L_1 to L_2.

In general, there is a clear need to treat inferences about the "*spatial aspects*" in an integrated manner with inferences about the "*causal aspects*" of a system; an endeavour, that we hypothesize can be achieved by explicitly representing both aspects, namely spatial and causal, within one representational framework. Using such an integrated approach, it is possible to infer *cause* from observed change or prescribe change (e.g., spatial re-configuration or planning) based on purpose, thereby serving as a goal-directed spatial-control mechanism in relevant application domains (e.g., intelligent robotic applications). For instance, a certain spatial transformation (or a sequence of transformations depending on the granularity) resulting in a particular spatial situation could characterise a desired goal-situation. Note, however, that inferring purpose from change/observations or prescribing change based on purpose is only possible if there is indeed a teleological aspect to the spatial changes being modelled per se. For instance, whereas there can be a telic aspect to the sequence of spatial changes determined by the *turn-actions* that a robot may undertake whilst following a route description, a telic-aspect is not applicable in a situation such as the following: "*The village was washed away in the tsunami*," where causation is applicable, but without a telic aspect. We maintain the hypothesis that the notion of causality is essentially primal to that of teleology, i.e., teleological phenomena can be necessarily modelled using a causal specification, but not vice-versa. Furthermore, we assume that whenever the teleological aspect of spatial changes needs to be exploited, there is indeed such an aspect to the spatial changes being considered, the modelling of which depends on the existence of a set of causal axioms that relate domain specific "*spatial occurrents*" to the underlying domain-independent spatial changes that are representable. Here, *spatial occurrents* refer to those events or actions that involve, either as a precondition or as a direct or indirect effect, some form of transformation or change over the domain-independent (relational) spatial structures being modelled. For example, whereas *turn-left* is a spatial occurrence since it effects the underlying orientation information, *paint-wall* is not a spatial occurrence, assuming colour is not regarded as a spatial attribute.

Situation Calculus as a Representational Formalism. A wide range of formalisms have been developed for the modelling of dynamically changing environments, e.g., situation calculus (McCarthy & Hayes, 1969), event calculus (Kowalski & Sergot, 1986), fluent calculus (Thielscher, 1998). The utility of such higher level representational formalisms (involving reasoning about actions and change) for the modelling of dynamic spatial systems cannot be taken granted—rather fundamental problems (e.g., Frame, Ramification, Qualification) relevant to modelling changing environments have been thoroughly investigated in the context of the class of formalisms aforementioned (Shanahan, 1997). This has resulted in several nonmonotonic extensions to classical symbolic approaches that are better suited for representing humanlike abilities of commonsense reasoning with incomplete information. Furthermore, the issue of concurrent and continuous phenomena, which manifest themselves even in the simplest of dynamic domains (both spatial and aspatial), has been rigorously investigated in the context of the class of formalisms developed within the area of reasoning about actions and change (Lin & Shoham, 1992; Reiter, 2001; Shanahan, 1997; Shoham, 1988). A dynamical systems perspective for spatial modelling in the context of these formalisms lends itself to well-founded representational and computational apparatus for dealing with commonly occurring problems with regard to time, continuity, concurrency and change.

Applicability of Formal Spatial Calculi. Ontological distinctions notwithstanding, the representational and computational aspects of arbitrary qualitative (spatial) calculi are based on common semantics. In this research, the high-level aspects of axiomatic spatial calculi relevant to differing aspects of space are of significance. We provide a step-by-step generalisation of the manner in which every aspect of a qualitative spatial calculus may be modelled using the proposed causal theory. The main advantage of the proposed formalisation of existing calculi is that fundamental computational tasks involving projection and explanation directly follow from the semantics of the formalisation. These tasks lie at the foundation of several application domains that involve modelling of humanlike spatial reasoning abilities in real and simulated environments, as control mechanisms in robotic/intelligent systems and within Geographic Information Systems (GIS) in the form of event-based explanatory models.

2. A SITUATION CALCULUS-BASED CAUSAL THEORY

The causal framework for modelling dynamic spatial systems is formalized using a customized version of the situation calculus formalism. The customization follows the requirements that are necessary in order to adopt a foundational notion of events that is causal in nature. The rich situation calculus ontology, its general mechanism to formalise change and the modelling of spatial dynamics using it lends itself to the fundamental application-centric reasoning tasks.

2.1. A Causal Notion of Events

The ontological status of events has been an issue of much discussion and debate among philosophers (Davidson, 1969; Kim, 1976; Pianesi & Varzi, 2000; Quine, 1960). According to Quine (1960), events are to be regarded (in a manner similar to objects) as spatiotemporal regions with at most one event occupying a given spatiotemporal region of space. Starting with the original region-based calculus in Clarke (1991), which has a spatiotemporal interpretation, this position has gained serious attention toward the development of mereotopological, "spatiotemporal" theories of space (Muller, 1998a, 1998b; Hazarika & Cohn, 2001). At the heart of these spatiotemporal theories lies the premise that space-time histories of events (occurrents) and objects (continuants) be accorded a primitive ontological status within the theory.

The notion of events that is applicable within this framework is causal in nature and is aimed at characterising explicit causal and (if applicable) teleological accounts of the evolution of a process. This view is based on an alternate view of events, where events are identified according to their causes and effects (Davidson, 1969). Davidson suggests the (parameterized) individuation of events (and possibly actions, which are a special species of events) and their description in terms of their causal relations, i.e., by their causes and effects. According to Davidson, "*The causal nexus provides for events a 'comprehensive and continuously usable framework' for the identification and description of events analogous in many ways to the space-time coordinate system for material objects*." It is only through such a causal framework that a precise characterisation of events, actions, causality and (causal) explanation can be provided. Note that Davidson's causal characterisation of events also implicitly provides an identity criterion for events (and actions). As Davidson puts it: '*Events have a unique position in the framework of causal relations between events in somewhat the same way objects have a unique position in the spatial framework of objects*' (Davidson, 1969, pg. 179). For Davidson, sameness of cause and effect is a more useful criterion than sameness of place and time, i.e., events are identical if and only if they have exactly the same causes and effects. Differences in their respective notions of events within a causal framework notwithstanding, Kim (1971) echoes similar sentiments in proposing that an adequate causal framework consisting of a well-defined ontological and logical account of events and related entities such as facts, conditions, states, processes and phenomena is necessary for an analysis of events in terms of the causal relations that exist between them. According to Kim: "*Any discussion of causation must presuppose an ontological framework of entities among which causal relations are to hold, and also an accompanying logical and semantical framework in which these entities can be talked about ... the adequacy of an analysis of causal relations may very much depend on the sort of ontological and logical scheme underlying the causal framework*." On the issue of causal laws, Davidson (1967) expresses the view that a *cause* is the sum total of the conditions positive and negative taken together, which being realized, the consequent invariably follows (Davidson, 1967). What essentially follows from this, and has indeed been interpreted as such by researchers in causal theories of action, is that *causes* correspond to sentences that express conditions of truth and all causal laws are instances of the universal conditional; in other words, a causal law is merely a material implication. As illustrated in section 2.2, this is precisely how causality is interpreted in this work.

Events within a Causal Framework. Events within a causal framework (i.e., events identified by their causes and effects) may be interpreted differently depending on the problem being addressed. In general, the following distinctions are applicable:

(1) Internal Events: Events that are internal to the system being modelled and which have an associated occurrence criteria are referred to as *internal events*. Internal events are deterministic in the sense that if the occurrence criteria for an internal event is satisfied, the event will necessarily occur.

(2) External events: Events that are external to the system and which occur arbitrarily are referred to as *external events*. By arbitrary, we mean that unlike internal events, occurrence criteria for these events are not available. As an example, consider a simulation of the queue at a bank teller: an event characterised by the arrival of a new customer at the end of the queue is something external to the simulation of the queue; the only certainty from a simulation perspective being that at some point, the event will necessarily occur. Practically, external events can be accounted for within the context of a dynamic planner/controller where the system can continuously interface with the external world to poll for the occurrence of such events.

(3) Nondeterministic Events or Actions: Actions are agent-centric (i.e., performed by an agent) and are therefore, by definition volitional or have a nondeterministic will associated with them. Simply, all preconditions for a given action may be satisfied and yet the agent may not perform the action. The distinction into actions is mainly applicable in scenarios where spatial reasoning abilities of real or simulated agents are being modelled, e.g., robotic control software.

Henceforth, we refer to internal and external events and actions as occurrences. In the specialised spatial reasoning domain, occurrences may be defined at two levels: (1) On the basis of a typology of the fundamental spatial changes, which the primitive entities within the spatial theory may undergo, e.g., *growth, shrinkage, splitting, merging, appearance, disappearance, rotation and movement* (Claramunt & Thériault, 1995). At this level, the only identifiable notion of an occurrence is that of a qualitative spatial

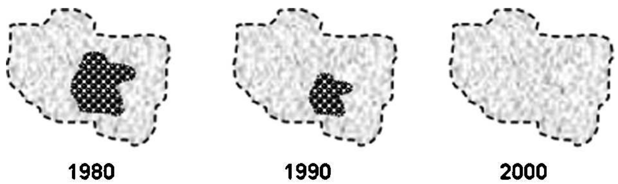

Figure 1. Shrinkage and disappearance.

transition that the primitive objects in the theory undergo. (2) Domain specific spatial occurrences (events or actions) that have (explicitly) identifiable occurrence criteria and effects that can be defined in terms of the fundamental typology of spatial change. For instance, in the example in Figure 1, we can clearly see that the contained/smaller region has continued to *shrink* over a 3 decade period, eventually *disappearing* altogether in the year 2000. In so far as a general theory of space or spatial dynamics is concerned, the only identifiable notion of *events* will be based on a primitive taxonomy of spatial change, i.e., in the example under consideration, the only identifiable events are *shrinkage* and *disappearance*. However, at a domain specific level, the observed phenomena can be causally related to *deforestation*, *fire* or other events. As such, at the domain-specific level, the following notion of a "*spatial occurrence*" is applicable—"spatial occurrences are either *events* or *actions* with explicitly specifiable *occurrence criteria* or *preconditions* respectively and *effects* that may be defined in terms of a domain independent taxonomy of spatial change that is native to a spatial theory. For example, a certain spatial event may *cause* a region to *split* into two or make it *grow/shrink*." Likewise, a spatial (control) action, e.g., *turn-left*, will have the effect of changing the orientation of the agent in relation to some other object. In certain situations, there may not be a clearly identifiable set of domain specific occurrences with explicitly known occurrence criteria or effects that are definable in terms of a typology of spatial change. However, even in such situations, an analysis of the domain independent events (e.g., event-based evolution of a process) may lead to an understanding of spatiotemporal relationships and help with hypothesis generation (Beller, 1991).

2.2. Explicit Notion of Causality

Causality is a vast topic and as a concept has aroused many debates and differing viewpoints from several quarters (Sosa & Tooley, 1993). The philosophical take on causality, i.e., "of or pertaining to the ultimate or true *cause* of things," is too powerful a notion to be applicable in practical domains. Given the practical nature of the problems addressed in this research, our understanding of causality is driven by the aim to support the modelling of temporal projection and explanation problems. The position on causality in research on causal theories of action has been that "causes" correspond to sentences that express conditions of truth and all causal laws are instances of the universal conditional. In other words, a causal law is merely a material implication. Here, causality is interpreted as some sort of a weak relationship that exists between known events or between events and properties of the system being modelled. Although researchers in causal logic maintain the distinction between being *true* and *necessarily* having a *cause*, the latter being, in the philosophical sense, something stronger than a material conditional, there is no way in the formal logical notation to express what precisely the *cause* may be (Giunchiglia et al., 2004; McCain & Turner, 1997). In the theoretical framework of this research, causality is interpreted either as a relationship of *direct dependence* between a known occurrence (event or action) and the state of affairs in the world or a temporally invariant "*indirect dependence*" between two sentences in the form of a state constraint. The distinctions are elaborated in the following:

(A) Direct Effects of Occurrences: The basic use of the causal relationship, as used in this work, is to represent the direct effects of occurrences, e.g., representing the fact that a certain domain specific event *causes* a region to *split*, *grow* or *shrink*. As aforediscussed, the primary aim in this paper is to modelling domain-independent spatial dynamics using a causal approach. As such, direct effects of domain-specific occurrences will not be dealt with here. In this paper, the only direct effects that are applicable are those relevant to modelling the effect of primitive spatial transitions (or changing qualitative spatial relationships) on the spatial fluents that are representative of distinct spatial domains being modelled.

(B) Indirect effects: State constraints constitute an important representational device in our work. As will be evident in the sections to follow, all aspects of a qualitative calculus can be represented using state constraints. However, (some) state constraints also pose serious problems such as containing indirect effects in them (Lin & Reiter, 1994; Lin, 1995). In the context of the situation calculus, Lin (1995) illustrates the need to distinguish ordinary state constraints from indirect effect yielding ones, the latter being also referred to as *ramification constraints*. This is because when ramification constraints are present, it is possible to infer new effect axioms (or simply effects) from explicitly formulated (direct) effect axioms together with the ramification constraints. Simply speaking, ramification constraints lead to what can be referred to as "*unexplained changes,*" which is clearly undesirable. In section 3.4.1 and 3.4.4, we illustrate the use of a causal relationship toward solving the problem of indirect effects that arises while modelling composition theorems within one spatial calculus and axioms of interaction between interdependent calculi.

2.3. Basic Formalism, Notation and Foundational Theory

The overall axiomatisation in the domain-independent causal theory, symbolically referred to as Σ_{causal}, consists of two main classes of axioms/formulae: (1) *Foundational Theory* (Σ_{sit}): This is the meta-level theory of the situation calculus and (2) *Spatial Theory* (Σ_{space}): Axiomatisation of the domain-independent spatial theory, which essentially formulates the underlying qualitative physics of the spatial domain being modelled. The following notation will be used throughout the paper.

Notation. We adopt the convention that all free variables are universally quantified from the outside and that the scope of all quantifications is limited to the respective sort of the variable being quantified. All variables as well as constants are represented using the lower-case alphabet. Where there is a possibility of ambiguity, we use letters with integral sub-scripts (e.g., o_1, o_2) to denote constants whereas those with generic ones (e.g., o_i, o_j) denote variables. Some notation (N1–N3) follows before we elaborate the key ingredients of the situation calculus formalism to be employed in this work and the foundational elements within Σ_{sit}:

N1. Fluents: Let $\Phi = \{\phi_1(\vec{x_1}), \phi_2(\vec{x_2}), \ldots, \phi_n(\vec{x_n})\}$ denote the set consisting of all propositional and functional fluents that collectively constitute the dynamical properties of the system being modelled.[1] The notation $\phi(\vec{x})$ is used as an abbreviation—given that $\{\vec{x} = [x_1, x_2]\}$, $\phi(\vec{x})$ is a shorthand for $\phi(x_1, x_2)$. Here, we do not imply that fluents may have a variable number of arguments; the arity is fixed and the abbreviation is simply a matter of convenience. Unless stated otherwise, all references to Φ henceforth refer to a subset of Φ that consists solely of *spatial fluents*, i.e., situation dependent spatial properties of the dynamic spatial system being modelled. Since we are primarily interested in a domain-independent spatial theory, it does not make sense to include domain specific aspatial properties, since these are inherently non-exhaustive and unknown.

N2. Fluent Values: Let $\Gamma_p = \{true, false\}$ consist of the possible denotations for propositional fluents and $\Gamma_f \equiv \{\gamma_1, \gamma_2, \ldots, \gamma_n\}$ consists of the denotation set for functional fluents in Φ. In general, we refer to the set of all possible fluent values as $\Gamma = \Gamma_p \cup \Gamma_f$. When necessary, we also use the notation ϕ_p or ϕ_f to indicate that ϕ is a propositional or functional fluent respectively.

N3. Occurrences: Let $\Theta = \{\theta_1(\vec{v_1}), \theta_2(\vec{v_2}), \ldots, \theta_n(\vec{v_n})\}$ represents the set of all apriori known *occurrences* within the system. Note that $\theta(\vec{v})$ has

[1]Note that fluents also have a time-dependent "situational" argument. Since this is not important at this point, the preceding characterisation does not mention the situational argument explicitly.

the same abbreviated interpretation as seen previously for $\phi(\vec{x})$. Each $\gamma_i \in \Gamma$ and $\theta_i(\vec{v_i}) \in \Theta$ has a specific interpretation and syntactic form in the context of a spatial theory Σ_{space}. These details, not being relevant at this point, are presented in the context of modelling the domain-independent spatial theory Σ_{space} in section 3.4. Note that both fluents as well as occurrences are typically parameterized. For instance, $\phi(o_1, o_2)$ could denote a spatial or aspatial relationship between two *objects* o_1 and o_2. Similarly, $\theta(o_1)$ could denote a spatial occurrence involving the manipulation of *object* o_1.

2.3.1. The Situation Calculus Language—$\mathcal{L}_{sitcalc}$. The situation calculus formalism used in this work is a first-order, many-sorted language with equality and the usual alphabet of logical symbols and their respective definitions. Henceforth, we refer to the situation calculus language used in this work as $\mathcal{L}_{sitcalc}$. A precise definition follows:

Definition 2.1 (The Language $\mathcal{L}_{sitcalc}$). *$\mathcal{L}_{sitcalc}$ is a first-order many-sorted language with equality and the usual alphabet of logical symbols—assuming their usual definitions, $\mathcal{L}_{sitcalc}$ uses the following symbols: $\{\neg, \wedge, \vee, \forall, \exists, \supset, \equiv\}$. Whereas the symbol $\equiv$ denotes semantic equivalence, additionally, the symbol $\equiv_{def}$ is used to define equivalence by definition. There are sorts for events and actions—Θ, situations—S, spatial objects—O and regions of space—R. Corresponding to each sort, there are countably infinite many variables for each sort that are respectively denoted as: (a) $\{\theta_1, \theta_2, \ldots, \theta_n\}$ for occurrences, (b) $\{s_1, s_2, \ldots, s_n\}$ for situations and (c) $\{o_1, o_2, \ldots, o_n\}$ for objects, and (d) $\{r_1, r_2, \ldots, r_n\}$ for regions of space. S_0 is a special symbol that denotes the initial situation.*

$\mathcal{L}_{sitcalc}$ consists of 5 foundational elements that are used in the formulation of the meta-theory Σ_{sit} and domain-independent spatial theory Σ_{space}. The intended interpretation for each of the foundational elements of $\mathcal{L}_{sitcalc}$ are elaborate in (L1–L5): ■

L1. Reified System Properties: A ternary predicate $Holds(\phi(\vec{x}), \gamma, s)$ denoting that fluent $\phi(\vec{x})$ has the denotation γ in situation s. Note that $\phi \in \Phi$. For clarity, we will use it in the following alternative ways: (a) Nonreified version: $[\phi(\vec{x}, s) = \gamma]$ and (b) Reified version: $Holds(\phi(\vec{x}), \gamma, s)$. Depending on what is more convenient, a nondeterminate situation is expressed in the following alternate ways using the *Holds* predicate:

$$[\phi(\vec{x}, s) = \{\gamma_1 \ \vee \ \gamma_2\}] \equiv_{def} [Holds(\phi(\vec{x}), \gamma_1, s) \vee Holds(\phi(\vec{x}), \gamma_2, s)] \tag{1}$$

L2. Precondition Axioms: Spatial theory specific possibility criteria for the *spatial transitions* that are identifiable in the spatial theory are specified using the binary predicate symbol $Poss(\theta, s)$, where $\theta \in \Theta$. $Poss(\theta, s)$

denotes that a spatial transition θ is *possible* in situation s. *Poss* is also used to represent the preconditions of domain-specific actions.

$$\begin{aligned} Poss(\theta(\vec{v}), s) \equiv [&Holds(\phi_1(\vec{x_1}), \gamma_1, s) \\ &\wedge \cdots \wedge Holds(\phi_n(\vec{x_n}), \gamma_n, s)] \end{aligned} \tag{2}$$

Equation (2) is the generic form for a precondition axiom. The intended interpretation here being that for occurrence $\theta(\vec{v})$ to be *possible* in situation s, fluents $\phi_1(\vec{x_1}), \ldots, \phi_n(\vec{x_n})$ should have the respective denotations of $\gamma_1, \ldots, \gamma_n$ in situation s. Note that it is not necessary that if an occurrence is possible in a given situation, it will necessarily happen in that situation.

L3. Event Occurrence Axioms: Recall the distinctions of occurrences into events and action. Whereas actions are agent-centric and therefore, volitional, events necessarily *occur* without any intervention when their occurrence criteria are satisfied.

$$\begin{aligned} [Holds(\phi_1(\vec{x_1}), \gamma_1, s) \wedge Holds(\phi_2(\vec{x_2}), \gamma_2, s)] \\ \supset Occurs(\theta(\vec{v}), s) \end{aligned} \tag{3}$$

Occurrence axioms of the form in (3) establish the occurrence criteria for events. Here, the intended interpretation is that if it is true $\phi_1(\vec{x_1})$ and $\phi_2(\vec{x_2})$ have the denotation γ_1 and γ_2 respectively in situation s, then event $\theta(\vec{v})$ will necessarily *occur* in situation s. The '*Occurs*' predicate is used to represent the knowledge about deterministic events within the system that are necessarily triggered by well-defined conditions that *hold* in the world.

L4. Effect Axioms: A ternary $Caused(\phi_i, \gamma, s)$ predicate, where $\phi_i \in \Phi$ and $\gamma \in \Gamma_i$, denoting that the fluent ϕ_i is *caused* to take on the value γ in situation s. The *Caused* relation, which essentially expresses a weaker notion of causality (see following sub-section), is used to represent the effects of occurrences in the following two ways: (a) *Direct effects*, where occurrences are directly stated to effect named fluents via effect axioms, (b) *Indirect effects*, where fluents take on values based on the satisfaction of some situation-specific criteria. As such, the *Caused* predicate is always a direct (direct effects) or indirect link (indirect effects) between fluents and occurrences.

L5. The "Result" Function: The binary function symbol [$Result : occurrence \times situation \rightarrow situation$] that denotes the unique situation resulting from the happening of an occurrence $\theta \in \Theta$ in a particular situation s.

2.3.2. The Foundational Causal Theory Σ_{sit}. On the basis of the language established in $\mathcal{L}_{sitcalc}$, the foundational theory or meta-level theory Σ_{sit} for the spatial modelling task needs to be defined. A precise definition follows:

Definition 2.2 (Foundational Theory Σ_{sit}). *The foundational theory Σ_{sit} of the situation calculus formalism consists of the following set of formulae: the property causation axiom determining the relationship between being 'caused' and being 'true', a generic frame axiom in order to incorporate the assumption of inertia, uniqueness of names axioms for the fluents in Φ, occurrences in Θ and fluent denotations in Γ, and the domain closure axioms for propositional ($\phi_p \in \Phi$) and functional fluents ($\phi_f \in \Phi$). Σ_{sit}, consists of the formulae comprising of:* (4), (5), (6), *and* (7). *The foundational aspects are discussed in (F1–F3).* ■

F1. Property Causation: For the predicate *Caused*, we need (4) denoting that if a fluent ϕ is *Caused* to take on the value γ in situation s, then ϕ holds the value γ in s. Note that a system property, spatial or aspatial, *holds* in a particular situation if and only if it is "*Caused*" to assume a certain denotation in that situation. To reiterate, this *causation* can only happen as a result of a direct or indirect effect of a known occurrence.

$$Caused(\phi(\vec{x}), \gamma, s) \supset Holds(\phi(\vec{x}), \gamma, s) \tag{4}$$

$$\begin{aligned} &\{Poss(\theta(\vec{v}), s) \vee Occurs(\theta(\vec{v}), s) \supset \\ &\quad [\neg(\exists\gamma')\ Caused(\phi(\vec{x}), \gamma', Result(\theta(\vec{v}), s)) \supset \\ &\quad Holds(\phi(\vec{x}), \gamma, Result(\theta(\vec{v}), s)) \equiv Holds(\phi(\vec{x}), \gamma, s)]\} \end{aligned} \tag{5}$$

F2. Noneffects of Occurrences: We also need to incorporate the noneffects of occurrences so that inertial properties may be propagated in future situations. For this purpose, we include a generic *frame axiom* (5) thereby incorporating the principle of inertia, i.e., unless "*Caused*" otherwise (either directly or indirectly), a fluent's value, i.e., its denotation in a particular situation, will necessarily persist. In principle, the generic frame axiom is needed in order to incorporate the commonsense law that unless proved otherwise, most system properties or fluent values remain the same when an event of actions occurs.

$$When\ i \neq j, [\theta_i(\vec{x}) \neq \theta_j(\vec{y})] \tag{6a}$$

$$[\theta_i(\vec{x}) = \theta_j(\vec{y})] \supset [\vec{x} = \vec{y}] \tag{6b}$$

$$When\ i \neq j, [\phi_i(\vec{x}) \neq \phi_j(\vec{y})] \tag{6c}$$

$$[\phi_i(\vec{x}) = \phi_j(\vec{y})] \supset [\vec{x} = \vec{y}] \tag{6d}$$

$$[true \neq false] \wedge [\gamma_1 \neq \gamma_2 \neq \cdots \neq \gamma_n] \tag{6e}$$

$$(\forall\ \gamma, s).\ [Holds(\phi_p(\vec{x}), \gamma, s)] \supset [\gamma = true \vee \gamma = false] \tag{7a}$$

$$(\forall\ \gamma, s).\ [Holds(\phi_f(\vec{x}), \gamma, s)] \supset [\gamma = \gamma_1 \vee \gamma = \gamma_2 \vee \cdots \vee \gamma = \gamma_n]$$
$$where\ every\ \gamma_i \in \Gamma_f \tag{7b}$$

F3. Unique Names and Domain Closure Axioms: We also need unique names axioms (UNA) for all occurrences $\theta_i(\vec{x}) \in \Theta$ as well as all fluents $\phi_i(\vec{x}) \in \Phi$. Additionally, similar axioms are needed for each of the potential denotations $\gamma_i \in \Gamma$ for propositional and functional fluents. (6a–6b) and (6c–6d) consist of generic uniqueness of names axioms for occurrences and fluents respectively. (6e) is the UNA for propositional and functional fluent values. Finally, we need the appropriate axioms in order to enforce the constraint that across all *situations*, the potential denotations for all fluents $\phi_i(\vec{x}) \in \Phi$ are closed under the denotation-set Γ. (7a) and (7) constitute the domain closure axioms for propositional and functional fluents, respectively.

3. Σ_{space}—THE DOMAIN INDEPENDENT SPATIAL THEORY

3.1. A Region Based Spatial Abstraction

We operate within a purely region-based framework involving spatially extended objects and maintain the typical ontological distinction between an object and the region of space that it occupies, with "*Space*" being used as the transfer function from the domain of *spatial objects* (O) to the domain of *regions* (R). Whenever necessary, the transfer function can be used to make the necessary distinctions. Pragmatically, a distinction between an object and its spatial extension will be of use only when the spatial theory is being used either in conjunction with an elaborate theory of physical objects or in a practical application where the precise spatial interpretation for an *object* is obtained by way of a separate module that performs the relevant geometric manipulations (i.e., computing convex-hulls, minimal-bounding rectangle, minimal convex polygon, voronoi partitioning etc). When there is no ambiguity, we refer to spatial relationships as directly holding between *objects* of the domain instead of their spatial extensions. For this purpose, the object-region equivalence axiom in (8) is included—depending on context, we refer to spatial relationships as directly holding between *objects* of the domain instead of their spatial extensions.

$$\begin{aligned} Holds(\phi_{spatial}(o_1, o_2), \gamma_1, s_1) \equiv_{def} [(\exists r_i, r_j)\ space(o_1, s_1) = r_i\ \wedge \\ space(o_2, s_1) = r_j \wedge Holds(\phi_{spatial}(r_i, r_j), \gamma_1, s_1)] \end{aligned} \tag{8}$$

The region-based framework, as used in this work, is suitable for fine-scale analysis with primitive objects or macro-level analysis with aggregates of entities (with dynamic physical properties) that have a well-defined spatiality (see section 3.2). However, as will be demonstrated in section 3.4, the underlying causal approach for the modelling of dynamic spatial systems is equally applicable for the modelling of spatial dynamics within a (spatial) framework consisting of different ontological commitments, e.g., spatial calculi involving point or line-segment based primitive objects.

3.2. Primitive Objects and Complex Aggregates

Some assumptions regarding the nature of the regions within the domain-independent spatial theory Σ_{space} are essential. This is primarily in order to preserve the generality of the theory for fine-scale analysis with primitive entities or macrolevel analysis with aggregates or clusters of entities. Furthermore, the notion of "region validity" that we employ in Definition 3.1 within the theory is partly influenced to ensure compatibility with the requirements of the typical region-based calculi such as in the spatial domain. Validity of regions within the theory is defined as follows:

Definition 3.1 (Valid Regions within the Theory). *A region is valid if it has a well-defined spatiality, is measurable using some notion of n-dimensional measurability that is consistent across inter-dependent spatial domains and if the region is convex and of uniform dimensionality. We elaborate each of the assumptions and their resulting implications in the context of the spatial theory in (A1–A4):* ■

A1. Well-Defined Spatiality: Regions in the theory correspond to the spatial extents of *objects*, with an object denoting a primitive physical entity or aggregate entity (some collection of objects) that has a well-defined spatiality. The latter scenario is typical of applications in the GIS area, e.g., spatial/temporal analysis in epidemiology, wildlife biology, or the study of diffusion processes in general, where the underlying domain consists of aggregate or clusters of geospatial entities.

A2. Measurability: The size of a region is equivalent to the size the object, which we assume can be defined using some notion of its n-dimensional measure, e.g., an object measurable in R^n could have size defined as its length $(1D)$, area $(2D)$ or volume $(3D)$.

A3. Consistency across Spatial Domains: The particular interpretation for a region and the notion of its n-dimensional measure has to be *consistent* with regard to the inter-dependent spatial domains being used. For instance, when the available data is qualitatively mapped into the theory, the spatial interpretation for a region, the topological relationships between regions and their corresponding relative sizes (n-dimensional measures) should be consistent with each other.

A4. Convexity and Regularity: Finally, we assume that the spatial extensions of objects are regular, convex regions of space that approximate the object in question, e.g., using a convex hull primitive or a minimal bounding rectangle for primitive objects or a minimal convex polygon for aggregates of objects, the precise geometrical technique being applied is not relevant to our work. In so far as the dynamic spatial modelling task is concerned, the proposed causal theory is equally applicable in scenarios where the regions being modelled are nonconvex. Strictly speaking, this assumption may or may not be applicable depending on the richness of the spatial calculus being modelled.

On the basis of the assumptions (A1–A4), it is possible to define an appropriate spatial semantics for modelling phenomena such as *growth* and *shrinkage* for *regions* which do not have a truly spatial manifestation. For instance, consider a typical GIS scenario involving a temporal snap-shot where a set of sample *points* (e.g., GPS-based locations) are mapped on the two-dimensional surface, which for simplicity can be assumed to be the Euclidean plane R^2. Irrespective of what these points represent, one objective could be to establish clusters of these points by using some nearest neighbour heuristic, following which, the obtained clusters may be interpreted as two-dimensional regions of space on the basic of their minimal convex-hulls. Since the convex polygons are measurable sets in R^2, phenomena such as *growth*, *shrinkage* would directly correspond to *growth*, *shrinkage* in its spatial extent. Such a well-defined spatiality for the regions is important if information relevant to different aspects of space has been used in an integrated manner (see section 3.4.4).

3.3. Dynamic Object Properties

Objects in the domain may have varying properties relevant to their physical aspects at different times. To aid the discussion, lets appeal to a commonsense notion of rigidity where objects tend to maintain their shape; this is essentially similar to the physics based notion where a rigid body is an idealization of a solid body of finite size in which deformation is completely neglected. In other words, the distance between any two given points of a rigid body remains constant regardless of external forces exerted on it. Given this interpretation, an important issue that concerns the characterisation of dynamic object properties is that of classification of objects into “strictly rigid” and “non-rigid” types. Consider the following scenarios:

(SC1) A “delivery object” (o) is *disconnected* (dc) ‘*next to*’ a delivery vehicle (v) in one situation (s_1) and in a later situation (s_2), is *inside* the delivery vehicle. Topologically, this is equivalent to the following: (i) Situation s_1: $Holds(\phi_{top}(o, v), dc, s_1)$, (ii) situation s_2: $Holds(\phi_{top}(o, v), tpp, s_2)$.

(SC2) A *container* object is completely filled with *water*. In this state, the container (or water) can still contain some other object, let's say, by way of *dropping* a small metal *ball* in the *container*. Now lets say that in a later situation, the *water* is frozen and stays that way for eternity.

When dealing with material (rigid) objects, such as the metal ball in scenario 2, the observed topological changes can be understood to be the result of motion, rather than other possibilities such as continuous deformation that are possible with non-rigid objects, such as fluids. However, a coarse distinction into strictly rigid and nonrigid objects is not sufficient. For example, consider the delivery vehicle (or the room) in the examples aforementioned. Although the object identifying the vehicle cannot *grow* or *shrink*, it can certainly contain other objects. Therefore, the vehicle can neither be classified as being strictly rigid (being in a similar class as that of a metal ball), thereby not allowing interpenetration, nor is it a fully flexible nonrigid object like a water body that can *grow*, *shrink* or change *shape*. To take the case further, the solidification of the water-body in scenarios 2 reveals that upon its being frozen, there is a fundamental change in the physical property of water. This change, namely water being solidified into ice, is important and must be reflected as a change of spatial (physical) property from a fully flexible to a strictly rigid object so that the container, which was previously filled with water and could still contain other objects cannot contain other objects anymore. Finally, an issue worth pointing out pertains to the dimensionality of fully-flexible bodies such as water or *fluids* in general. It is interesting to note that such bodies (strictly speaking, the dimensionality of the region of space occupied by such bodies) can also assume a dynamic form, i.e., such bodies assume the dimensionality of the "containing" object. An elaborate characterisation of the ontological issues pertaining to the nature of objects and their dynamic physical properties is not central to our work. We are primarily interested in a generic notion of such dynamic physical properties and the constraints on the potential spatial transformations that follow.

Dynamic Physical Properties and Constraints. In order to support the inclusion of dynamic physical properties of the sort mentioned before, we include a generic notion of such properties in the following:

Definition 3.2 (Dynamic Physical Property). *A dynamic physical property is that which characteristically pertains to the physical nature of a material object and which necessarily (dynamically) restricts the range of spatial relationships that the respective objects, or class of objects, can participate in with other objects, or class of objects. In general, we refer to the set of dynamic physical properties as* $\Phi_{physical} = \{\phi(o_i), \phi(o_j), \ldots, \phi(o_n)\}$. *These properties inherently pertain to individual objects (unary), are dynamic (i.e., modelled as fluents), have a propositional denotation and constitute a part of the overall spatial fluent set, i.e.,* $[\Phi_{physical} \subset \Phi]$. ■

Example 3.1 (Containment and Deformation Properties). *For exemplary purposes, two trivial distinctions can be directly applied on the basis of the containment and deformation properties of objects. Let O and S refer to the set of domain-objects and situations respectively. The dynamic physical properties consists of the following:*

- $allows\_containment \subseteq [O \times S]$*—Propositional fluent denoting that a given object may contain other objects in a particular situation.*
- $can\_deform \subseteq [O \times S]$*—Propositional fluent denoting that a given object may continuously deform by way of* growth, shrinkage*, or change of* shape.
- $rigid(o, s) \equiv_{def} [\neg allows\_containment(o, s) \wedge \neg can\_deform(o, s)]$
- $non\_rigid(o, s) \equiv_{def} [allows\_containment(o, s) \wedge can\_deform(o, s)]$ ■

A comprehensive characterization of dynamic physical properties is enormous, if not infinite, and is dependent on the spatial domain being modelled. It is not our objective to attempt a detailed classification of object/property categories and the kind of changes permissible therein.

Definition 3.3 (Dynamic Physical Constraint). *Dynamic physical constraint expresses a temporally invariant dependency between a physical and a spatial property by limiting the potential spatial relationships that the particular domain object may assume with other existing objects. Let $\phi_{physical}(o_i)$ be a propositional fluent characterizing a dynamic physical property for an object o_i as per Definition 3.2. Let $\vec{o}$ denote a collection of n objects such that it also includes o_i and let $\phi_{space}(\vec{o})$ be a n-ary spatial relationship relevant to some spatial aspect between the n domain objects given by $\vec{o}$. Finally, let $\Gamma_{space} = \{\gamma_1, \gamma_2, \ldots, \gamma_n\}$ represent a subset of the overall potential denotation set for the spatial relationship ϕ_{space}. A dynamic physical constraint is of the syntactic form given in* (9)*:*

$$(\forall\ s).\ \{Holds(\phi_{physical}(o_i), true, s) \supset \neg[Holds(\phi_{space}(\vec{o}), \gamma_1, s) \vee Holds(\phi_{space}(\vec{o}), \gamma_2, s) \vee \cdots \vee Holds(\phi_{space}(\vec{o}), \gamma_n, s)]\} \tag{9}$$

■

Note that like physical properties, dynamic physical constraints of the form in (9) are definable only within a specific spatial framework. For instance, containment constraints can be identified within the context of a mereotopological framework. Similarly, constraints on the potential rotation and direction of motion of objects (e.g., by *turn* and *move* actions) can be defined within a spatial framework consisting of orientation and direction information. It is essential that both dynamic physical properties as well as constraints be modelled at the level of a domain-independent spatial theory. This way, domain-independent constraints on the potential spatial transformations can be defined and used by modellers in arbitrary spatial scenarios. Before we exemplify dynamic constraints in Example 3.2, the following notion of the physical consistency of a situation is essential:

Definition 3.4 (Physical Consistency). *A situation is physically consistent if it satisfies all dynamic constraints relevant to every dynamic physical property.* ■

Example 3.2 (Rigidity and Non-Rigidity Constraints). *Given the dynamic physical properties of containment and deformation in Example 3.1, and the generic notion of a dynamic physical constraint in Definition 3.3, the following constraints on the potential topological transitions for the following categories of objects are definable: (a) Fully flexible nonrigid objects, which is the general case* (10a)*, (b) Combination of rigid and nonrigid Objects* (10b)*, (c) Semirigid and rigid objects* (10c)*, and (d) Strictly rigid objects* (10d)*.*

$$\begin{aligned}&(\forall o, o')(\forall s)\ [\mathit{non\_rigid}(o, s) \wedge \mathit{non\_rigid}(o', s)\\ &\qquad \supset \mathit{Holds}(\phi_{top}(o, o'), \gamma, s)] \\ &\mathit{where}\ \gamma \in \{dc, ec, po, eq, tpp, ntpp, tpp^{-1}, ntpp^{-1}\}\end{aligned} \tag{10a}$$

$$\begin{aligned}&(\forall o, o')(\forall s)\ [\mathit{rigid}(o, s) \wedge \mathit{non\_rigid}(o', s) \supset \mathit{Holds}(\phi_{top}(o, o'), \gamma, s)]\\ &\mathit{where}\ \gamma \in \{dc, ec, po, eq, tpp, ntpp\}\end{aligned} \tag{10b}$$

$$\begin{aligned}&(\forall o, o')(\forall s)\ [\mathit{allows\_containment}(o', s) \wedge \neg \mathit{can\_deform}(o', s)\\ &\qquad \wedge\ \mathit{rigid}(o, s) \supset \mathit{Holds}(\phi_{top}(o, o'), \gamma, s)] \\ &\mathit{where}\ \gamma \in \{dc, ec, po, eq, tpp, ntpp\}\end{aligned} \tag{10c}$$

$$\begin{aligned}&(\forall o, o')(\forall s)\ [\mathit{rigid}(o, s) \wedge \mathit{rigid}(o', s) \supset \mathit{Holds}(\phi_{top}(o, o'), \gamma, s)]\\ &\mathit{where}\ \gamma \in \{dc, ec\}\end{aligned} \tag{10d}$$

■

Similar properties can be identified for other spatial domains such as orientation, in which case constraints can be based on the integration of more than one spatial domain. For example, objects may have an intrinsic *front, rear, left, right, top, base/bottom* on the basis of which constraints may be specified: "two objects can only be *connected* from their respective *left* sides" or when one object enters (i.e., containment) another one, 'the intermediate *external connection* and *partial overlap* can only happen via the latter's *intrinsic front*'. Note that constraints can also be specific when the primitive spatial entities being modelled are not spatially extended. For example, in

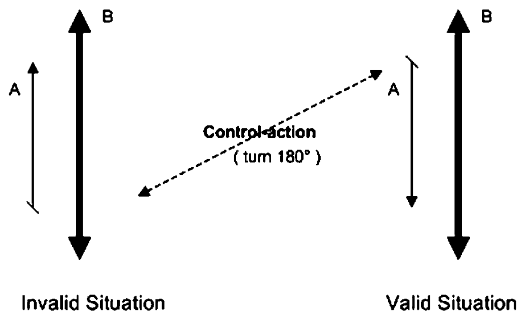

Figure 2. Dynamic constraints and control actions.

Figure 2, "*A delivery vehicle (A) and a* way-station *(B) can only be aligned to each other with respect to a certain (mutual) orientation,*" as a result of which it will be necessary for the *vehicle* to execute a 180° '*turn*' action in order to satisfy the given constraint if it is incorrectly oriented in relation to the way-station. Here, actors/objects are conceived as directed line segments, which is common within a line-segment based framework such as the Dipole Calculus (Moratz et al., 2000). Note that the constraints such as these or the ones in (10a–10c) essentially form a part of the spatial theory and exist independently of the domain being modelled. This is important in order to enforce a clear separation between a domain independent spatial theory and a domain specific axiomatisation that utilises the general theory.

3.4. Modelling Qualitative Spatial Calculi with Σ_{sit}

A study of qualitative spatial calculi from the viewpoint of their formal algebraic properties (e.g., (Ligozat & Renz, 2004)) is not relevant in this work. Only the high-level aspects of axiomatic spatial calculi pertaining to different aspects of space such as topology (e.g., region connection calculus (Randell et al., 1992)), orientation (e.g., line-segment based dipole calculus (Moratz et al., 2000), point-based double-cross calculus (Freksa, 1992a, 1992b) that are ubiquitous within the qualitative spatial reasoning domain are of significance in this work. Ontological distinctions pertaining to the nature of primitive spatial entities (regions, points or line-segments) notwithstanding, these spatial calculi are based on similar axiomatic semantics—precisely, these consist of a finite set of jointly exhaustive and pair-wise disjoint (JEPD) relations, compositional inference and consistency maintenance and the representation of change on the basis of the continuity of the underlying relation space, i.e., based on the conceptual neighbourhood principle (Freksa, 1991). Note that the primitive that make up the qualitative relation space could be of arbitrary arity, e.g., binary relations denoting topological relationships of the region connection calculus, ternary relations of the point-based doublecross calculus etc.[2] Furthermore, when more than one spatial domain (e.g., topology, orientation, size) is being used in a nonintegrated manner, we assume that appropriate axioms of interaction between such interdependent aspects are explicitly provided. This is because when interdependent spatial domains are used in a nonintegrated manner, spatial relationships from one domain entail the other and vice-versa. For instance, topological and size relations are not independent from each other—some topological relations entail size relations and vice-versa (Gerevini & Renz, 2002). Similarly, another form of interdependence, which is compositional in nature, occurs when topological and intrinsic-orientation relationships between spatially extended objects interact. As will be elaborated in section 3.4.4, we assume that when multiple interdependent aspects of space are being used in a non-integrated manner (i.e., they have separate sets of composition theorems), their axioms of interaction will be explicitly specified using an appropriate scheme that is suited to representing such interdependent entailments.

Notation for the Spatial Theory—Σ_{space}: At least in so far as this theory is concerned, there exist the following 3 types of spatial fluents—$\Phi = \Phi_{space} \cup \Phi_{physical} \cup \Phi_{exist}$: (1) Situation specific spatial relationships between objects (Φ_{space}), (2) Dynamic physical properties of objects ($\Phi_{physical}$), and (3) Existential properties of objects that determine whether an object *exists* in a particular situation (Φ_{exist}). Depending on the spatial domains being covered, there is one fluent for each type of spatial relationship between the primitive objects of the domain. For instance, assuming spatially extended objects with intrinsic orientation, one possible instantiation involves non-integrated usage of topological, orientational and size relationships: $\Phi_{space} = \{\phi_{top}, \phi_{ort}, \phi_{size}\}$. Furthermore, each $\phi_i \in \Phi_{space}$ has a finite denotation set, e.g., $\Gamma_{top} = \{dc, ec, po, eq, tpp, ntpp, tpp^{-1}, ntpp^{-1}\}$. Recall from the earlier discussion that in so far as a spatial theory is concerned, the only applicable notion of an occurrence is that of a primitive spatial transition definable in it. A *spatial transition* refers to a change of qualitative spatial relationship between the entities in the domain. $\Theta \equiv \{\theta_1, \theta_2, \ldots, \theta_n\}$ denotes the set of all spatial theory specific primitive *spatial transitions* in the theory. Each $\theta \in \Theta$ takes the form of $tran(\theta, o_i, o_i)$, read as o_i and o_j transition to the state of being in relation θ.

Definition 3.5 (Spatial Theory Σ_{space}). *The spatial theory Σ_{space} consists of the formalisation of an underlying domain-independent qualitative physics using the foundational causal theory Σ_{sit}. Σ_{space} is based on the abstract notion of a qualitative spatial calculus and consists of a systematic axiomatisation of all aspects relevant to modelling one or more (possibly interdependent) spatial calculi.* ■

3.4.1. Composition Theorems as Ramification Constraints. A straight-forward way to represent every composition theorem is to model it as an ordinary state

[2]For brevity, we assume binary spatial relationships in all subsequent formulae/examples.

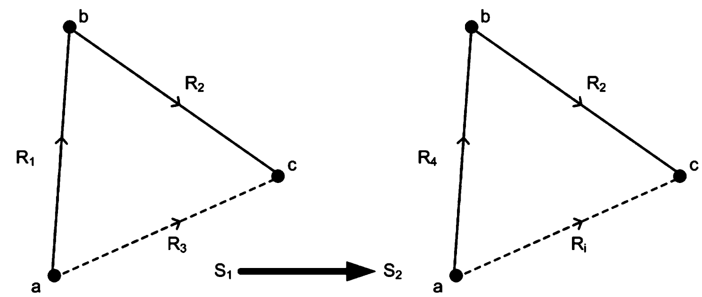

Figure 3. Compositional constraints and ramifications of spatial transitions.

constraint (11a), which is a standard way to represent temporally invariant facts about the logical relationship between fluents. However, as discussed in section 2.2, modelling composition theorems in this manner leads to unexplained changes since the resulting constraints contain indirect effects in them. For instance, Figure 3 illustrates a two trivial constraint networks, referred to as S_1 and S_2 in the discussion to follow. Here, nodes represent objects, whereas edges represent spatial relationships among them. Assuming that *situations* correspond to an instantaneous snapshot of the world, S_1 and S_2 correspond to 2 spatial *situations* such that $S_1 < S_2$.[3] With this setup, note that each constraint network essentially establishes a consistency criteria on situations S_1 and S_2, i.e., in situation S_1, $R_1(a, b)$ holds and so forth. Now, let's suppose that the relationship between a and b undergoes a *transition* to the conceptually neighbouring qualitative state of R_4, i.e., in situation S_1, a and b are related by R_1 whereas in situation S_2, they are related by R_4. Also, assume that the relationship between b and c remains the same. In order to satisfy the compositional consistency criteria, following the composition $[R_4 \circ R_2]$, imposed by the underlying relational space on situation S_2 (see second constraint network in Figure 3), the resulting change of relationship between a and c must be *explainable* within the underlying theory. Note that similar indirect effects also arise when interdependent spatial domains (e.g., topology and size) are utilized with their respective compositional constraints, i.e., in a nonintegrated manner. Here, such effects are a result of the mutual entailments between the interdependent spatial domains (see section 3.4.4).

Following the general treatment of indirect effects by Lin and Reiter (1994), we represent all ramification or indirect effect yielding constraints by utilizing an explicit notion of *causality* (introduced in section 2.2) via the ternary *Caused* relation. Using this scheme, we will need total of 8×8

[3]The ordering relation on *situations*, in the context of the situation calculus, is nontrivial and has a well-defined semantics (Reiter, 1993). This, however, is not important here in the present context.

constraints of the form in (11b).

$$(\forall s)\ Holds(\phi(o_1, o_2), \gamma_1, s) \wedge Holds(\phi(o_2, o_3), \gamma_2, s) \supset Holds(\phi(o_1, o_3), \gamma_3, s) \tag{11a}$$

$$(\forall s).\ Holds(\phi(o_1, o_2), \gamma_1, s) \wedge Holds(\phi(o_2, o_3), \gamma_2, s) \supset Caused(\phi(o_1, o_3), \gamma_3, s) \tag{11b}$$

This way, by minimising the extensionality of the *Caused* predicate whilst keeping the background foundational (Σ_{sit}) and spatial theory (Σ_{space}) fixed for every relevant *situation*, causation axioms determining precisely the fluents that undergo a change (either directly or indirectly) as a result of named occurrences can be derived. The (circumscriptive) minimisation policy applied in this framework is highlighted at the point of actual application in section 3.4.5.

3.4.2. Continuity Constraints of Relation Space. In the context of a qualitative theory of spatial change, the most primitive means of change is an explicit change of spatial relationship between two objects (their spatial extensions). To re-iterate from the notation for the spatial theory, let $tran(\gamma, o_i, o_j)$ denote such a change, read as, o_i and o_j *transition* to a state of being γ. The possibility axiom for such a transition has been formally expressed in a general manner in (12).

$$Poss(tran(\gamma, o_i, o_j), s) \equiv [\{space(o_i, s) = r_i \wedge space(o_j, s) = r_j\} \wedge \{(\exists\ \gamma')\ Holds(\phi(r_i, r_j), \gamma', s) \wedge neighbour(\gamma, \gamma')\}] \tag{12}$$

The binary predicate $neighbour(\gamma, \gamma')$ in (12) is used to express the possibility of a direct continuous transition (deformation or motion) being consistent between two spatial relations and is based on the conceptual neighbourhood principle (Freksa, 1991). According to this principle, relations γ and γ' are conceptual neighbours if two objects related by γ can directly transition to the state of being γ' and vice-versa. The conceptual neighbourhood graph for a particular set of n spatial relations can be used to define a total of n axioms of the form in (12) so as to comprehensively represent the possibility criteria for every definable spatial transition. However, for these continuity constraints to be meaningful, we also need the adequate definitions for the symmetric predicate *neighbour*. Note that *neighbour* is simply an auxiliary predicate instead of being a situation-dependent fluent. This is because the continuity structure that is being modelled always remains the same.

3.4.3. Direct Effect Axioms. The direct effects of named occurrences need to be explicitly specified within the axiomatisation. Occurrences could be either domain specific events and actions or domain-independent *spatial transitions* that represent a change of qualitative spatial relationship between the primitive spatial entities of the domain. For every spatial transition that is possible given the underlying relational space, we need direct effect axioms of the form in (13a).

$$\begin{gathered}\theta = tran(\gamma_1, o_1, o_2) \wedge \; Poss(\theta, s) \supset \\ Caused(\phi_{space}(o_1, o_2), \gamma_1, Result(tran(\gamma_1, o_1, o_2), s))\end{gathered} \tag{13a}$$

$$\begin{gathered}Occurs(\theta(\vec{o}), s) \vee Poss(\theta(\vec{o}), s) \supset \\ (\exists\gamma).\; Caused(\phi(\vec{o}), \gamma, Result(\theta(\vec{o}), s))\end{gathered} \tag{13b}$$

Here, since the focus is on a domain-independent spatial theory, the axiomatisation is restricted to the domain-independent case of "spatial transitions." However, note that the approach to model the direct effects remains the same for domain-specific events and actions. For instance, compare (13a) and (13b) for the respective cases—here, (13b) is the generic form for the representation of direct effects of domain-specific occurrences.

3.4.4. Axioms of Interaction between Interdependent Calculi. Axioms of interaction are only applicable when more than one spatial domain is being modelled in a non-integrated manner. By the use of more than one spatial domain, we refer to the composition of spatial relations pertaining to two different aspects of space in order to yield a spatial relation of either of the spatial types used in the composition. For instance, size *equality* rules out all containment (*tpp*, *ntpp* and their inverses) relationships. Similarly, if it is known that object o is a tangential part of object o', then it can also be presumed that the size of object o is less than the size of o'. Similar dependencies also exist between topological and (intrinsic) orientation relationships. For instance, consider the composition of topological and orientation relations *front* and *inside* involving 3 objects A, B, and C in (14). Here, it is clear that the *contained* object (C) has the same orientation relationship with other objects (A) as the containing object (B). Hence a topological relation implies an orientation relation.

$$\begin{aligned}(\forall o_a, o_b, o_c, s).\; & space(o_a, s) = r_a \wedge space(o_b, s) = r_b \; space(o_c, s) = r_c \wedge \\ & Holds(\phi_{ort}(r_a, r_b), front, s) \wedge Holds(\phi_{top}(r_c, r_b), inside, s) \supset \\ & Caused(\phi_{ort}(r_a, r_c), front, s)\end{aligned} \tag{14}$$

Axioms of interaction of the form in (14) provide an explicit characterisation of the relative entailments that exist between interdependent aspects of space. Note that the entailments may be nondeterminate; however, they will still need to be explicitly axiomatised in the form in (14). Based on the composition constraints in section 3.4.1 and axioms of interaction, the following notion of compositional consistency is important from a computational viewpoint (see section 4):

Definition 3.6 (Compositional Consistency). *A situation is compositionally consistent if it satisfies all the composition constraints of every spatial domain being modelled as well as the relative entailments among interdependent spatial calculi when more than one domain is being utilised.* ■

3.4.5. Causal Laws of the Spatial Theory. Successor state axioms (SSA) specify the causal laws of the spatial theory being modelled, i.e., what changes as a result of the occurrences in the system being modelled. Generally, the SSA is based on a *completeness assumption* which essentially means that all possible ways in which the set of fluents may change is explicitly formulated, i.e., there are no indirect effects (Reiter, 1991); we refer to this SSA as the Pseudo successor state axiom (PSA). The SSA that needs to be derived here, referred to as SSA-Proper, must also account for indirect effect yielding state constraints. Recall the use of the causal relation $Caused(\phi, \gamma, s)$ in (11b) toward the representation of the composition table theorems and axioms of interaction in addition to direct effects. Before we go into the details of SSA derivation, the following notion of a causation axiom is essential:

Definition 3.7 (Causation Axiom). *Given a domain theory consisting of: (a) explicitly formulated direct effects, (b) ramification constraints expressed using the ternary Caused relation, (c) ordinary domain constraints expressed using the Holds predicate, (d) the generic frame axiom and (e) unique names and domain closure axioms for fluents, occurrences and fluent values, a causation axiom for a fluent $\phi \in \Phi$ universally characterizes all the direct and indirect effects of named occurrences.* ■

What remains to be done is to minimize the causal relation by circumscribing (or using some other form of minimisation) it with the following set of axioms fixed—the foundational axioms in (4–7), the ramification constraints of the form in (11b) (i.e., compositional constraints and axioms of interaction) and the transition preconditions of the form in (12). The result of minimization is the *Causation Axiom* in (15).

Proposition 3.1 (Causation Axiom Derivation). *Given the background theory $\Sigma_{sit} \cup \Sigma_{space}$, the minimisation of the ternary Caused relation using circumscription yields the causation axioms as defined in Definition 3.7.* (15) *is the generic form of the required causation axiom. Notice that following the two ways, as proposed in section 2.2, in which the Caused relationship is used in this work,* (15a) *accounts for the direct effects on the fluent ϕ,* *whereas* (15b) *accounts for the indirect effects on* ϕ.

$$Caused(\phi(o_i, o_k), \gamma, s) \equiv (\exists s').\ s = Result(tran(\gamma, o_i, o_k), s') \wedge Poss(tran(\gamma, o_i, o_k), s') \wedge \neg Holds(\phi(o_i, o_k), \gamma, s') \tag{15a}$$

$$\begin{aligned} Caused(\phi_1(o_i,\ o_k), \gamma_k, s) \equiv\ & [\{(\exists o_j, \gamma_i, \gamma_j)\ Holds(\phi_1(o_i, o_j), \gamma_i, s) \wedge \\ & Holds(\phi_1(o_j, o_k), \gamma_j, s)\} \vee \\ & \{(\exists\ \gamma_l)\ Holds(\phi_2(o_i, o_k), \gamma_l, s)\}] \\ & where\ \phi_1, \phi_2 \in \Phi_{space} \end{aligned} \tag{15b}$$

Proof. This proposition is based on well-known results in Lin and Reiter (1994) and McIlraith (2000) for a general class of causal theories that utilise a primitive causal relationship for representing the direct and indirect effects of events and actions. A proof sketch follows:

Firstly, note that ϕ_1 and ϕ_2 in (15b) are spatial relationships representative of differing and complementary or interdependent spatial domains, i.e., $\phi_1, \phi_2 \in \Phi_{space}$, where $\Phi_{space} \subset \Phi$. Also, note that:

1. The basic use of the ternary *causal* relationship is solely toward representing the direct effects of occurrences (13a) and for modelling ramification yielding constraints (11b).
2. The ternary '*Caused*' predicate occurs only on the right-side of the material implication connective '$\supset$' in (13a) and (11b), which in the present context has a causal interpretation.

Given (1) and (2), in deriving the causation axiom of the form in (15), the extensionality of the 'Caused' predicate is minimized to include only those effects that are directly or indirectly explainable by the background theory and the set of facts that hold in a given situation. This is achieved by the application of a circumscriptive approach that transforms the material implication of the form in (13a) and (11b) to an equivalence of the form in (15a) and (15b)—this involves a syntactic transformation that follows from a standard result in circumscription (Lifschitz, 1994, pg. 5).

Intuitively, it is simple to see that when the '*Caused*' predicate is minimized, its extension (with regard to a specific spatial fluent $\phi \in \Phi_{space}$) includes only those *effects*[4] that have been explicitly included in the axiomatisation—these invariably consist of the following: direct effects (15a) and indirect effects (15b) of occurrences. ■

The causation axiom in (15) needs to be utilized in conjunction with the PSA, which is the SSA without indirect effects, to derive the real successor

[4]Model theoretically, this means instantiations for its variables.

state axioms for each fluent. The PSA in (16a) is derived from the generic frame axiom in (5) and the property causation axiom in (4):

$$\begin{aligned} &Poss(\theta, s) \supset [Holds(\phi(o_i, o_j), \gamma, Result(\theta, s)) \equiv \\ &\quad \{Caused(\phi(o_i, o_j), \gamma, Result(\theta, s))\} \vee \\ &\quad \{Holds(\phi(o_i, o_j), \gamma, s) \wedge \\ &\quad \neg\, (\exists \gamma').\ Caused(\phi(o_i, o_j), \gamma', Result(\theta, s))\}] \end{aligned} \tag{16a}$$

The causation axioms in (15) must be integrated with the (PSA) (16a) to derive the SSA-Proper in (16b), which accounts for both direct as well as indirect effects.

$$\begin{aligned} &Poss(\theta,\ s) \supset [Holds(\phi_1(o_i, o_j), \gamma_i, Result(\theta, s)) \equiv \\ &\quad \{(\forall\ \gamma')\ Holds(\phi_1(o_i, o_j), \gamma_i, s) \wedge \theta \neq tran(\gamma', o_i, o_j)\} \vee \\ &\quad \{\theta = tran(\gamma_i, o_i, o_j)\} \vee \\ &\quad \{(\exists o_k, \gamma_j, \gamma_k)\ Holds(\phi_1(o_i, o_k), \gamma_j, Result(\theta, s)) \wedge \\ &\quad Holds(\phi_1(o_k, o_j), \gamma_k, Result(\theta, s))\} \vee \\ &\quad \{(\exists\ \gamma_l)\ Holds(\phi_2(o_i, o_j), \gamma_l, Result(\theta, s))\}] \\ &\qquad where\ \phi_1, \phi_2 \in \Phi \end{aligned} \tag{16b}$$

To re-iterate, the effect of minimising the causal relation is to derive the causation axioms that essentially includes contextual conditions (direct or indirect) that will *cause* a fluent's value to change. These causation axioms are then compiled with the PSA in order to obtain the SSA-Proper.[5]

3.4.6. JEPD and Other Properties. The property of the spatial relationships being jointly exhaustive and mutually disjoint can be expressed using ordinary state constraints of the form such as in (11a). In general, we need a total of n state constraints of the form in (17a) to express the jointly-exhaustive property of n base relations.

$$\begin{aligned} &(\forall s).\ \neg[Holds(\phi(o_1, o_2), \gamma_1, s) \vee Holds(\phi(o_1, o_2), \gamma_2, s) \\ &\qquad \vee \cdots \vee Holds(\phi(o_1, o_2), \gamma_{n-1}, s)] \supset \\ &\qquad\qquad Holds(\phi(o_1, o_2), \gamma_n, s) \end{aligned} \tag{17a}$$

$$(\forall s).\ \neg[Holds(\phi(o_1, o_2), \gamma_1, s) \wedge Holds(\phi(o_1, o_2), \gamma_2, s)] \tag{17b}$$

[5] We have included a generic overview because of space restrictions. However, a step-by-step illustration of the general strategy in the context of indirect effects of actions can be found in Lin and Reiter (1994) and Lin (1995).

$$(\forall s).\ [Holds(\phi(o_i, o_j), \gamma, s) \supset Holds(\phi(o_j, o_i), \gamma, s)] \tag{18a}$$

$$(\forall s).\ [Holds(\phi(o_i, o_j), \gamma, s) \supset \neg Holds(\phi(o_j, o_i), \gamma, s)] \tag{18b}$$

Similarly, $[n(n-1)/2)]$ constraints of the form in (17b) are sufficient to express the pairwise disjointness of n relations. Additionally, other miscellaneous properties such as the symmetry (18a) and asymmetry (18b) of the base relations too can be expressed using ordinary constraints.

3.5. Appearance and Disappearance of Objects

Appearance of new objects and disappearance of existing ones, either abruptly or explicitly formulated in the domain theory, is characteristic to dynamical systems. In robotic applications, it is necessary to introduce new objects into the model, since it is unlikely that a complete description of the robot's environment is either specifiable or even available. Similarly, it is also typical for a mobile robot operating in a dynamic environment, with limited perceptual or sensory capability, to loose track of certain objects because of issues such as noisy sensors or a limited field-of-vision. Such behaviour, involving the modification of the domain of discourse, is not unique to applications in robotics. Even within event-based geographic information systems, such events are regarded to be an important typological element for the modelling of dynamic geospatial processes (Claramunt & Thériault, 1995; Worboys, 2005). For instance, Claramunt and Thériault (1995) identify the basic processes, used to define a set of low-order spatiotemporal events which, among other things, include appearance and disappearance events as fundamental.

Representational and Computational Difficulties. The difficulty of modelling of such behaviour emanates from the general problem of the nonmodifiability of the underlying domain of discourse and is rooted in the structure and semantics of model-theory. Consequently, problem is not unique to situation calculus but rather occurs with every formalisation that utilises a logic-based or model-theoretic semantics. Toward the representation of event-operators that lead to the appearance and disappearance of domain objects, Gooday and Cohn (1996) identify a similar problem in the context of transition calculus, which is a high-level formalism for reasoning about action and change (Gooday & Galton, 1997). Drawing on the methodology adopted by Gooday and Cohn, we illustrate our solution to this problem, at least in so far as the present spatial modelling task in the context of the situation calculus formalism is concerned. A general solution, if even possible, is beyond the scope of this work.

3.5.1. Foundational Extensions to Σ_{space}. Several extensions at the foundational level are needed in order to support the appearance and disappearance of objects. In addition to the inclusion of a new unary propositional fluent, namely *exists*(o), to denote the existential status of an object, we also include two special external events, namely *appearance* and *disappearance*, that *directly* effect this new fluent. The causal laws in (19a–19b) formalise the direct effects of *appearance* and *disappearance* events on the existential status of an object within the domain theory. Furthermore, the possibility axioms for the appearance and disappearance events are defined in terms of the existential status of objects. This is formalised using the occurrence pre-condition axioms in (19c–19d). Note that the two special events need to be introduced at the foundational level and must be regarded to be external in nature, i.e., no criteria or conditions determining the occurrence of these special events is available. These special events are usable by an arbitrary domain in order to dynamically introduce new objects or remove existing ones from the model on the basis of domain specific criteria.

$$(\forall o, s).\ [Occurs(disappears(o), s) \supset \\ Caused(exists(o), false, Result(disappears(o), s))] \tag{19a}$$

$$(\forall o, s).\ [Occurs(appears(o), s) \supset \\ Caused(exists(o), true, Result(appears(o), s))] \tag{19b}$$

$$(\forall o, s).\ [Poss(disappears(o), s) \equiv Holds(exists(o), true, s)] \tag{19c}$$

$$(\forall o, s).\ [Poss(appears(o), s) \equiv Holds(exists(o), false, s)] \tag{19d}$$

$$(\forall o, s).\ [Occurs(disappears(o), s) \supset \\ (\forall o').\ Caused(\phi_{space}(o, o'), null, Result(disappears(o), s))] \tag{19e}$$

$$(\forall o, s).\ [Occurs(appears(o), s) \supset \\ (\forall o').\ Caused(\phi_{space}(o, o'), null, Result(appears(o), s))] \tag{19f}$$

A special *null* relationship that may *hold* between objects (i.e., their spatial extents) is also required. The causal laws in (19) formalise the direct effects of *appearance* and *disappearance* events on the spatial relationship of a new and existing object (respectively) on other objects. These causal laws merely postulate that the appearance or disappearance of any *object* in any *situation* will *cause* the spatial relationship, given by ϕ_{space}, to assume a *null* denotation. Note that the *null* relationship (or symbol so to speak) is ontologically elevated to the status of a qualitative label, similar to any other qualitative spatial relationship, within the set of spatial relationships that form the vocabulary of the spatial calculus that is being modelled. As such, a qualitative calculus with n JEPD relationships is in actuality modelled within the present setup as one with $n + 1$ JEPD relationships. The following notion of existential consistency is essential and compliments the previously introduced definitions of physical consistency and compositional consistency.

Definition 3.8 (Existential Consistency). *A situation is existentially consistent if there exists at least one non-null spatial relationship that every existing spatial object participates in with other existing objects.* ■

The approach that we use draws on the work by Gooday and Cohn (1996), where Gooday and Cohn use a state-based approach that is much like a STRIPS style system with *add* and *delete* lists and *change-operators*. Their approach is centered around the us of a forward and backward completion mechanism for the propagation of facts about the existence of objects and their spatial relationship with other objects into the state-based history of the system being modelled. Similar to their approach, we maintain existential facts about objects at the foundational level, with the difference that in our approach, such facts have been (implicitly) temporalized. Additionally, our approach differs with respect to the manner in which we integrate appearances and disappearances and the resulting spatial relationships between objects. Since the existential facts about objects are temporalized, and their is no notion of a global-state, it is also not necessary to propagate the facts backwards or forward into the *situation* based history since such facts are by definition situation-dependent. Therefore, an object can continue to *exist* until a certain point in the systems evolution (i.e., till a particular *situation*) and then cease to exist, after which it cannot enter into any spatial relationship with existing objects. For situations *before* the situation in which the new object appears, a situation-specific minimization (using circumscription[6]) of the *Holds* predicate will lead to the desired *implicit* assumption of the non-existence of the new object in the past situations. Finally, we do not presuppose that the spatial relationship(s) that a newly emerging object has with (at least) one or (possibly) more existing objects is explicitly provided. As such, using this approach, it is not necessary to know the spatial relationships of the new object with existing ones. By default, the new object has a special *null* relationship with other objects. However, in subsequent situations, and in domain specific ways, the new object's spatial relationship(s) with other objects is updated when new information becomes available.

3.6. Initial State of the World—Big Bang Situation

The final part of the spatial theory consists of a description of the *initial state* of the dynamic system being modelled. The initial state (henceforth *situation*) description corresponds to the situation in which no occurrences have happened. This is the standard big-bang situation typically referred to

[6]As with minimization of the indirect effects, the minimisation of the extensionality of the *Holds* predicate too is achieved using circumscription. This is done for consistency of approach; however, in the latter case, simple Clarke completion or a default closed world assumption too may be applicable.

as S_0 in the literature on situation calculus. In so far as the spatial properties of the system are concerned, the initial state description is either partially or completely specifiable. Note that by complete specification, we do not imply absence of uncertainty or ambiguity. Completeness also includes those instances where the uncertainty is expressed as a set of completely specified alternatives, typically in the form of disjunctive information.

Fluent Categorisation. The initial situation, henceforth referred to as S_0, basically includes a specification of initial fluent values. The fluents can be broadly categorised in spatial fluents and aspatial fluents. Aspatial fluents consist of domain-specific dynamic properties that are not spatial in nature. As such, these are not applicable in the present context of the spatial theory. For spatial fluents (Φ), there exist *three* broad categories of fluents:

(1) Existential Facts (Φ_{exists}): These are propositional fluents that provide an explicit existential characterisation of every spatial object that is known to exist in the initial situation. It is necessary for every known object to *exist* in the initial situation. For a domain consisting of n objects in the initial situation, n facts of the form '$Holds(exists(o), true, S_0)$' are required.

(2) Spatial Relationships (Φ_{space}): These model the spatial relationships that exist between the objects that are known to exist in the initial situation. For every type of spatial relationship being modelled, the *initial situation* (S_0) description involving n domain objects requires a complete $n-$ *clique* specification with $[n(n-1)/2]$ spatial relationships of one type or spatial domain. As noted previously, this may either be specified explicitly or can be derived, albeit with a certain level of uncertainty, from the explicitly provided partial specification. Note that in S_0, none of the objects can be related by a *null* spatial relationship.

(3) Dynamic Physical Properties ($\Phi_{physical}$): characterise the dynamic object properties relevant to their physical characteristics.

Partial Description and Monotonic Extension. When spatial relationships (Φ_{space}) between some objects are omitted, a complete description (with disjunctive labels) can be derived on the basis of the composition theorems for the spatial domain under consideration. The following notion of a 'monotonic extension' is necessary:

Definition 3.9 (Monotonic Extension). *Let Ω denote a partial spatial situation description consisting of facts expressed using the ternary Holds predicate. $O \equiv \{o_i, o_j, \ldots, o_n\}$ denotes the set of spatial entities and $\Phi^{arity}_{type} \in \{\phi^1_{exists}, \phi^2_{space}, \phi^1_{physical}\}$ denotes the set of spatial relationships that may exist between the spatial objects. The monotonic extension of a partial spatial situation description Ω is another description Ω' of a similar nature such that all semantic entailments with respect to the spatial information present in Ω*

*are preserved in Ω'. This monotonicity condition holds if the following criteria are satisfiable: (i) $\Omega \subset \Omega'$, and **IF**: $\Omega[s] \models (\exists \vec{o}, \gamma).\ [Holds(\phi(\vec{o}), \gamma, s)]$ **THEN**: $\Omega'[s] \models [Holds(\phi(\vec{o}), \gamma, s)]$* ■

Condition (i) in Definition 3.9 stipulates that Ω' contains additional facts than are present in Ω. Condition (ii) expresses that none of the (spatial) entailments of Ω' invalidate or refute the information contained in Ω. Note that the notation $\Omega[s]$, read as the result of substituting the *situational* argument in all instances of the *Holds* predicate to situation s (i.e., s is the only free-variable in the Ω).

Proposition 3.2 (Compositional Consistency of $\Omega \cup \Omega'$). *The conjunction of a partial situation description with its monotonic extension Ω' is compositionally consistent as per the notion of compositional consistency in Definition 3.6.*

Proof. This directly follows from the fact that Ω' is derivable, under the standard provability relation '$\vdash$', from the facts present in Ω and the composition theorems (Σ_{CT}; see 11b) and axioms of interaction (Σ_{INT}; see 3.5.4) contained within the spatial theory Σ_{space}. Consider the simplest case in (20a–20b) involving topological and size relationships between 3 objects O_1, O_2 and O_3—here, Ω denotes the initial partial description involving the 3 objects.

$$\begin{aligned} \Omega \ \equiv \ & [Holds(\phi_{top}(o_1, o_2), tpp, S_0) \wedge Holds(\phi_{top}(o_2, o_3), dc, S_0) \ \wedge \\ & Holds(\phi_{size}(o_2, o_3), =, S_0)] \end{aligned} \tag{20a}$$

$$\begin{aligned} & \Omega \ \wedge \ \Sigma_{CT} \wedge \Sigma_{INT} \vdash \Omega', \ \textit{where} \\ \Omega' \ \equiv \ & [Holds(\phi_{top}(o_1, o_2), tpp, S_0) \wedge Holds(\phi_{top}(o_2, o_3), dc, S_0) \ \wedge \\ & Holds(\phi_{top}(o_1, o_3), dc, S_0) \wedge Holds(\phi_{size}(o_1, o_2), <, S_0) \ \wedge \\ & Holds(\phi_{size}(o_2, o_3), =, S_0) \wedge Holds(\phi_{size}(o_1, o_3), <, S_0)] \end{aligned} \tag{20b}$$

Given Ω, Ω' can be monotonically derived ($\vdash$) on the basis of Ω and the RCC-8 composition theorems (Σ_{CT}) and axioms of interaction between topology and size (Σ_{INT}).[7] Here, Ω' is a monotonic extension of Ω in the sense that while new information is conjoined with Ω, none of the existing spatial knowledge is invalidated. ■

[7]Note that the completion of *Holds* predicate is obtained on a situation-by-situation basis. This is necessary in order to obtain a syntactically correct, but semantically equivalent, complete situation description, i.e., the actual monotonic extension includes twice as many spatial relationships as are included in a complete n-clique description.

On the basis of Proposition 3.2, it is also easy to see that the monotonic extension of a partial situation description is physically as well as existentially consistent, as per their respective definitions in 3.4 and 3.8. This is because the only new facts that are added in the monotonic extension pertain to the spatial relationships between objects.

4. SPATIAL REASONING WITH $\Sigma_{sit} \cup \Sigma_{space}$

Given the structure and semantics of the situation calculus based axiomatisation of the causal theory $[\Sigma_{sit} \cup \Sigma_{space}]$, fundamental reasoning tasks involving projection and explanation can be directly represented. Within the specialised domain of dynamic spatial systems, these essentially translate to spatial property projection and explanation, which then can be used for modelling useful reasoning tasks involving spatial planning and/or reconfiguration, causal explanation of dynamic spatial phenomena and qualitative spatial simulation.

4.1. Spatial Planning

Given the background spatial theory (i.e., one or more spatial domains modelled using the causal approach), domain specific constraints, an initial state and an overall objective to be achieved, one task is to derive a sequence of (spatial) actions that will fulfil the desired objective. In other words, how do we transform one spatial configuration into another? Or alternately, what are the spatial transformations that are necessary corresponding to the achievement of a certain goal? Here, a goal can be a situation in which a certain action has happened or where some fluents hold specific values. Note that this problem, which can be considered akin to the task of arriving at a desired spatial configuration starting at an initial configuration, is one of the simplest form of spatial planning. Variations along this line involve the incorporation of dynamically available information (e.g., sensing-abilities of a robot) in the planning process, since an incremental plan generation approach, where sensing affects subsequent planning, is more powerful in comparison to an off-line or static approach. In the following, we investigate the general structure and semantics of a basic spatial planning task.

Spatial Property Projection. In the context of the causal theory Σ_{sit} and Σ_{space}, specialised notions of spatial property projection and spatial situation legality need to be defined on the basis of the general projection and legality testing tasks. These are necessary in order to provide a formal account of the reasoning tasks that follow from the axiomatisation in Σ_{sit} and Σ_{space}. Furthermore, in conjunction with previously defined notions of the physical, existential and compositional consistency of a spatial situation, this is also necessary develop a formal account of *physically realizable* spatial situations, namely those situations that can actually be physically realised on the basis of the underlying qualitative physics specified in accordance with the causal theory $[\Sigma_{sit} \cup \Sigma_{space}]$.

Before we define *physical realizability*, we formally present the notion of spatial property projection and spatial situation legality testing. The projection of a spatial property is defined as follows:

Definition 4.1 (Spatial Property Projection). *Let $\vec{\theta} = [\theta_1, \theta_2, \ldots, \theta_n]$ denote a sequence of spatial events or actions. $\Phi[s] \equiv_{def} [\phi_1(\vec{o_1}) \wedge \phi_2(\vec{o_2}) \wedge \cdots \wedge \phi_n(\vec{o_n})]$ denotes the conjunction of spatial fluents with one free variable of the situation sort and all other variables bound to domain objects of the sort spatial object. S_0 denotes the initial situation when no occurrences have happened and Ω corresponds to a* valid (3.1) *initial situation description as defined in section 3.6.*[8] *The projection of $\Phi[s]$ corresponds to determining whether or not $\Phi[s]$ holds in the situation that results in the sequential application of the events and actions in $\vec{\theta}$, starting in the initial situation S_0. This is formally expressed in* (21)*:*

$$\Sigma_{sit} \cup \Sigma_{space} \cup \Omega \models (\exists s).\ s = Result(\vec{\theta}, S_0) \supset \Phi[s] \tag{21a}$$

$$\Sigma_{sit} \cup \Sigma_{space} \cup \Omega \models \{(\exists\ s).\ s = Result(\vec{\theta}, S_0) \supset [Holds(\phi_1(\vec{o_1}), \gamma_1, s) \wedge Holds(\phi_2(\vec{o_2}), \gamma_n, s) \wedge \cdots \wedge Holds(\phi_n(\vec{o_n}), \gamma_n, s)]\} \tag{21b}$$

$$Result(\vec{\theta}, s) \equiv_{def} Result(\theta_n, Result(\theta_{n-1}, \ldots, Result(\theta_2, Result(\theta_1, S_0)) \ldots)) \tag{21c}$$

Note that given the reified nature of 'relationships' throughout this work, it is not hard to see that the (21) is the correct syntactic transformation for the abbreviated form in (21a). Also, observe that $Result(\vec{\theta}, S_0)$ is actually an abbreviation for (21c). ■

Spatial Situation Legality. Clearly, spatial property projection alone is insufficient from the viewpoint of spatial planning, where one is interested in only those projections that are actually achievable given the constraints imposed by the action pre-condition and event occurrence axioms for the respective actions and events in $\vec{\theta}$ (21c). The following notion of the 'legality' of situations is necessary:

Definition 4.2 (Spatial Situation Legality). *A spatial situation is legal if either it is the initial situation S_0 or the situation resulting from an occurrence*

[8] Recall that this primarily consists of a complete n-clique description involving $[n(n-1)/2]$ spatial relationships between n domain objects.

whose pre-conditions or occurrence criteria are satisfied in a legal situation. This legality criteria is formally expressed in (22)*. Note that 'Legal(s)' is used as an abbreviation.*

$$Legal(s) \equiv_{def} (\forall \theta, s').\ [Result(\theta, s') \leq s] \supset [Poss(\theta, s') \vee Occurs(\theta, s')] \tag{22}$$

■

A spatial situation is '*valid*' if it is physically 3.4, existentially 3.8, and compositionally 3.6 consistent. Since we are interested in spatial projections, it is necessary this notion of situation validity be made stronger by combining it with the notion of the *legality* of a spatial situation in Definition 4.2. The result is the concept of a physically realizable situation:

Definition 4.3 (Physical Realizability). *A spatial situation is physically realizable if: (a) the 'existence' of all objects that participate in spatial relationships with each other is formally derivable, given that appearances and disappearances may have occurred (i.e., existential consistency* (3.8)*), (b) the objects satisfy the constraints relevant to their dynamic physical properties (i.e., physical consistency* (3.4)*), and (c) the spatial relationships that hold in the situation satisfy the compositional constraints of the underlying relation space (i.e., compositional consistency* (3.6)*), (d) The 'state' corresponding to the situation under consideration strictly consists of spatial fluents of the class mentioned in (a), (b), and (c), and (e) Finally, except for the initial situation S_0, every intermediate situation-term within the entire history should be the result of a spatial occurrence.*

Proposition 4.1 (S_0 is Physically Realizable). *The initial situation S_0 is physically realizable.*

Proof. This is a trivial base case that follows from the fact that: (a) The monotonic extension of a partial initial situation description is *valid* (section 3.6). (b) The initial situation is the situation where no occurrences have happened. As such, by definition, the initial situation S_0 is also *legal*. Consequently, S_0 is also physically realizable since no additional constraints relevant to legality testing apply. ■

Proposition 4.2 (Legality and Physical Realizability). *Situation legality is necessary and sufficient to ensure physically realizability, i.e., situation legality satisfies all constraints imposed by the underlying domain-independent (qualitative) physics modelled by way of the spatial theory Σ_{space}. In other words, if a spatial situation is legal, then it is also physically realizable.*

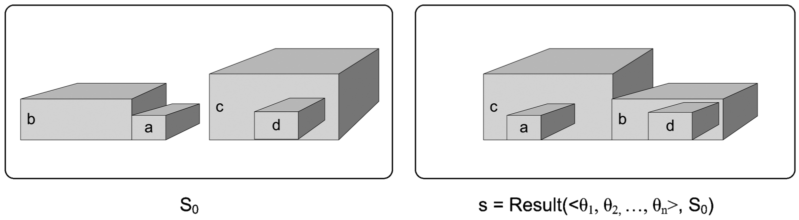

Figure 4. Spatial reconfiguration.

Proof. The main criteria that determines the physical realizability of a spatial situation is the satisfiability of all domain-independent constraints and pre-conditions as imposed by the underlying qualitative physics that is being modelled.

From the base case in Proposition in 4.1, we know that the initial spatial situation S_0 is legal as well as physically realisable. Given the construction of spatial situation legality in Definition 4.2, it is clear that the class of legal situations is a subset of the overall situational space, rooted in the initial situation S_0, where inclusion is defined on the condition that every such situation be the result of an occurrence whose preconditions are satisfied in a legal situation, starting with the initial situation S_0. Furthermore, given the causal laws of the domain as stipulated by the successor state axioms in 16b, it is clear that the spatial situation description that corresponds to a legal spatial situation also satisfies the existential and compositional constraints of the underlying relational space. Therefore, it is implied that if a situation is legal, then the continuity of the underlying relationship space and the overall global consistency—compositional, existential, and physical—is satisfied. Consequently, all legal situations are also physically realisable. ■

Example 4.1 (A Spatial Reconfiguration Task). *Spatial reconfigura-tion is a form of spatial planning where the objective is to derive a sequence of spatial transitions that will achieve the desired objective; here, an objective is specifiable by a desired configuration of the objects of the domain. For instance, given the following: the domain-independent spatial theory in Σ_{space}, the foundational axioms of the situation calculus in Σ_{sit}, a (partial) initial situation and a goal-state description in Ω_{ini}* (23a) *and $\Omega_{goal}[s]$* (23b) *respectively*[9] *(see Figure 4), the re-configuration task essentially involves*

[9]For illustration purposes, we utilizes the RCC-8 fragment of the region connection calculus (Randell et al., 1992) to represent topological information in addition to a simple intrinsic orientation system with labels *left*(l), *front*(f), *front* $-$ *left*(fl), and so forth.

deriving the entailment in (23c)[10]*—what needs to be done is to derive a legal-binding for the (only) free situation term s in* (23c) *as a side-effect of a theorem-proving task; this approach, where plans are synthesized as a side effect of theorem-proving being a standard account of planning in the situation calculus (Reiter, 2001).*

$$\Omega_{ini} \equiv [Holds(\phi_{top}(a,b), ec, S_0) \wedge Holds(\phi_{top}(d,c), tpp, S_0) \wedge Holds(\phi_{top}(a,c), dc, S_0) \wedge Holds(\phi_{top}(b,c), dc, S_0) \wedge Holds(\phi_{ort}(a,c), r, S_0) \wedge Holds(\phi_{ort}(b,a), r, S_0)] \tag{23a}$$

$$\Omega_{goal}[s] \equiv [Holds(\phi_{top}(a,c), tpp, s) \wedge Holds(\phi_{top}(d,b), tpp, s) \wedge Holds(\phi_{top}(b,c), ec, s) \wedge Holds(\phi_{ort}(c,b), r, s)] \tag{23b}$$

$$\Sigma_{causal} \cup \Omega_{ini} \models [(\exists s).\ Legal(s) \wedge S_0 \leq s \wedge \Omega_{goal}[s]] \tag{23c}$$

$$\left\{\begin{array}{l} s = Result(\langle\theta_{61}, \theta_{62}, \theta_{63}, \theta_{64}, \theta_{65}\rangle, Result(\vec{\theta_5}, \\ \quad Result(\vec{\theta_4}, Result(\vec{\theta_3}, Result(\vec{\theta_2}, Result(\vec{\theta_1}, S_0)))))) \\ \vec{\theta_1} = [tran_{11}(rf, a, c), tran_{12}(f, a, c)] \\ \vec{\theta_2} = [tran_{21}(po, d, c), tran_{22}(ec, d, c), tran_{23}(f, d, c)] \\ \vec{\theta_3} = [tran_{31}(fl, d, b), tran_{32}(f, d, b)] \\ \vec{\theta_4} = [tran_{41}(ec, d, b), tran_{42}(po, d, b), tran_{43}(tpp, d, b)] \\ \vec{\theta_5} = [tran_{51}(ec, a, c), tran_{52}(po, a, c), tran_{53}(tpp, a, c)] \\ \vec{\theta_6} = [tran_{61}(rf, b, c), tran_{62}(f, b, c), tran_{63}(lf, b, c), \\ \quad tran_{64}(l, b, c), tran_{65}(ec, b, c)] \end{array}\right\} \tag{23d}$$

For simplicity, assume that all objects in Figure 4 always have their respective 'fronts' facing the same direction. Although a proof cannot be included here, it is worth highlighting that for this particular example, the binding for the free situational term 's' takes the form of a situation-based history (23d) *that is rooted in the initial situation 'S_0'—i.e., the derived sequence of spatial transitions achieves the desired reconfiguration.* ■

[10]'$S_0 \leq s$' denotes that s includes S_0 in its subhistory and *Legal*(s) is as per the definition in (22).

Note that in the spatial reconfiguration example, we are not concerned with the high-level composites/aggregates which the domain-independent transition sequences are collectively representative of—at higher-levels, these could be modelled as macros or some other construct. What is relevant here is the derivation of fine-scale (or the scale or granularity permitted by a particular instantiation of the spatial theory in Σ_{space}) sequences of primitive spatial transitions that will, in conformity with the underlying qualitative physics in Σ_{space}, achieve the desired objective. At higher-levels, aggregates of these transition sequences will represent domain-specific control primitives that must be executed by a real or simulated agent.

4.2. Causal Explanation

Causal explanation is the process of retrospective analysis by the extraction of an event-based explanatory model from available spatial data (e.g., temporally ordered snapshots). Indeed, the explanation is essentially an event-based history of the observed spatial phenomena defined in terms of both domain-independent and domain-dependent occurrences. Causal and, if applicable, telic accounts of a process being modelled are applicable in a diverse range of geospatial phenomena, such as movement of clusters of animals (wildlife biology), monitoring people-clusters in times of crisis on the basis of GPS-based positional information (e.g., emergency and disaster management and planning, defence modelling and simulation) and even in the geospatial analysis of the spread of diseases (epidemiology), where an event-based model can be extracted (or evolution of the phenomena be defined) on the basis of the typology of fundamental spatial changes. Additionally, causal analysis is also applicable in real-time surveillance systems where the occurrence criteria for domain-specific events/actions can be defined on the basis of certain, possibly incompletely known, spatial-configurations of the domain objects and/or the patterns of their dynamic evolution. Using the theoretical framework developed in this work, it is possible to explain spatial phenomena at a higher-level either in terms of domain-specific occurrences that *cause* the observed changes or alternatively, in a domain-independent manner on the basis of a fundamental typology of spatial change such as *splitting*, *growth*, *movement* etc.

Property and Occurrence Driven Explanations. In the specific context of the causal theory $\Sigma_{sit} \cup \Sigma_{space}$, explanation may be primarily performed along two fronts:

(1) Occurrence driven causal explanation, where an event and action based history can *sufficiently* explain an observed state (e.g., temporal snapshot(s)) of the system. This corresponds to the typical explanation task involving a problem such as the stolen car scenario,[11] the objective is to explain the abnormality of an observation in terms of an event or action that may have caused it. In this form of explanation, the explanations being sought are required to be in the form of event and action occurrences.

(2) Property driven causal explanation, where an apriori known narrative (i.e., an incomplete description of the evolution of the system) in conjunction with interpolated spatial information sufficiently characterizes the present or observed state of the spatial system. Here, the interpolation is performed on the basis of the underlying qualitative physics of the spatial domain(s) being modelled (i.e., their continuity and compositional constraints). This form of explanation is similar in nature to that investigated by Cohn and Hazarika (2001b), where an abductive approach for deriving complete space-time histories from partial observations is proposed.

Note that typically it is not possible to classify problems as being occurrence or property driven, with the general case demanding the use of a combination of both approaches.

Occurrence Driven Causal Explanation. Explanation, in general, is a converse operation to temporal projection essentially involving reasoning from effects to causes, i.e., reasoning about the past (Shanahan, 1989). An abductive approach to explanation, in the context of the situation calculus formalism, has been proposed by Shanahan (1993, 1997). In the following, we outline the structure of the causal explanation task in the context of Shanahan's abductive approach—the objective here is to draw a correspondence between the generalized structure of an abductive approach to explanation and its specialisation for the causal explanation task with [$\Sigma_{causal} \equiv \Sigma_{sit} \cup \Sigma_{space}$]: Let Σ_{causal} be the background theory. Φ is an observation sentence whose assimilation demands some explanation. Additionally, a set of predicates are distinguished as being *abducible* in order to avoid trivial explanations; this is characterized in the abduction policy η^*. It is essential that the explanation Δ must be in terms of predicates that have been designated as being *abducible* in η^*. Given this, the causal explanation task in the context of the causal theory Σ_{causal} is to find a formula Δ such that $\Sigma_{sit} \wedge \Sigma_{space} \wedge \Delta \models \Phi$. Finally, an approach is needed to incorporate the non-effects of events and actions thereby overcoming the frame problem. This is achieved by the use of a minimisation policy similar to the one used in the derivation of the causation axioms in section 3.4.5. Definition (4.4) formalises the commonly understood

[11]The stolen-car scenario is typically used as a example of explanatory reasoning or reasoning backwards from effects to causes. Specifically, the objective in this scenario is to explain the abnormality of the disappearance of the car in a situation where one would *typically* expect to find it. The example dates back at least to Kautz (1986).

notion of an abductive approach to explanation based on the minimisation of effects of occurrences for the causal explanation task in the context of Σ_{causal}.

Definition 4.4 (Causal Explanation). *A formula Δ is a causal explanation of Φ in terms of the abduction policy η^* and given the background causal theory Σ_{causal} and a circumscription policy that minimizes the predicate symbols in ρ^* and allows the predicates, constants, and function symbols in σ^* to vary if:*

1. $\mathsf{CIRC}[\Sigma_{sit} \wedge \Sigma_{space} \wedge \Delta; \rho^*; \sigma^*]$ *is consistent,*
2. Δ *mentions only predicates in η^*, and*
3. $\mathsf{CIRC}[\Sigma_{sit} \wedge \Sigma_{space} \wedge \Delta; \rho^*; \sigma^*] \models \Phi$ ■

Depending on the abduction policy η^* (i.e., the predicates that are nominated as being abducible), the explanation Δ can be either property driven or occurrence driven. However, as indicated before, typical usages will involve both forms of explanation.

Example 4.2 (Abducing Transitions and Appearances). *Consider the illustration in Figure 5—the situation-based history $\langle s_0, s_1, \ldots, s_n\rangle$ represents one path, corresponding to a actual time-line $\langle t_0, t_1, \ldots, t_n\rangle$, within the overall branching-tree structured situational space. Furthermore, assume a simple system consisting of objects 'a', 'b', and 'c' and also that the state of the system is available at time point t_i and t_j. Note that the situational-path and the timeline represent an actual as opposed to a hypothetical evolution of the system. From the viewpoint of this discussion, two auxiliary predicates, namely HoldsAt(ϕ, t) and Happens(θ, t), that range over* time points *instead of* situations *are needed to accommodate the temporal extensions required to map a path in the situation-space to an actual time-line; complete definitions can be found in Pinto (1994). Given an initial situation description as in Φ_1 (see* (24)*), where 'b' does not exist and 'a' and 'c' are partially overlapping, in order to explain an observation sentence such as Φ_2, a formula of the form*

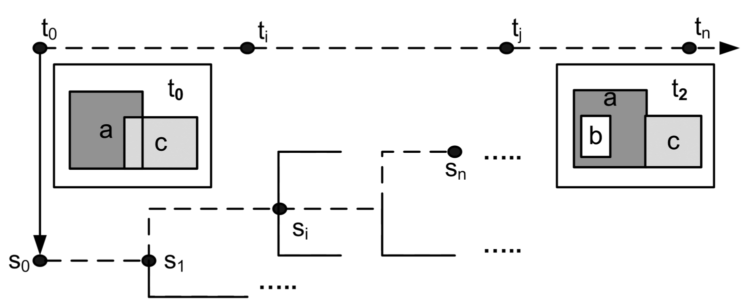

Figure 5. Abductive explanation.

in Δ needs to be derived.

$$\left\{\begin{array}{l} \Phi_1 \equiv HoldsAt(\phi_{top}(a,c), po, t_1) \\ \Phi_2 \equiv HoldsAt(\phi_{top}(a,c), ec, t_2) \wedge HoldsAt(exists(b), true, t_2) \\ \qquad\qquad \wedge\, HoldsAt(\phi_{top}(b,a), ntpp, t_2) \\ [\Sigma_{sit} \wedge \Sigma_{space} \wedge \Phi_1 \wedge t_1 < t_2 \wedge \Delta] \models \Phi_2,\ \textit{where} \\ \Delta \equiv (\exists t_i, t_j, t_k).[t_1 \leq t_i < t_2 \wedge Happens(appearance(b), t_i)] \\ \wedge\, [t_i < t_j < t_2 \wedge Happens(tran(b,a,tpp), t_j)] \wedge \\ [t_k < t_2 \wedge Happens(tran(a,c,po), t_k)] \wedge [t_k \neq t_i \wedge t_k \neq t_j] \end{array}\right\} \tag{24}$$

$$\mathsf{CIRC}[\Sigma_{sit} \cup \Sigma_{space} \cup \Phi_1 \cup t_1 < t_2 \cup \Delta; \rho^*; \sigma^*] \models \Phi_1 \tag{25}$$

The derivation of Δ primarily involves nonmonotonic reasoning in the form of minimising change ('Caused' and 'Happens' predicates), in addition to making the usual default assumptions about inertia. In order to show that the explanation Δ in (24) *is valid, it only needs to be shown that the observation is logically entailed by the circumscription of the background theory and the explanation taken together, i.e.,* (25) *needs to be established as per the notion of an explanation in Definition 4.4.* ■

5. DISCUSSION AND OUTLOOK

By regarding spatial theories as a specialisation within a higher-level framework to reason about change in general, spatial theories can be directly utilised in application domains involving reasoning about dynamic spatial phenomena. We have developed a causal theory for modelling dynamic spatial systems. The theory essentially utilizes a dynamical systems perspective for modelling spatial change, and primarily involves a step-by-step illustration of the manner in which different aspects of axiomatic spatial calculi may be accounted for within the causal framework. This is done with the aim to leverage upon the fundamental reasoning tasks that follow for the proposed axiomatisation.

Other foundational approaches toward the broader integration of spatial and logic-based common-sense reasoning frameworks is taken in the works of Allen and Ferguson (1994), Shanahan (1995), and Bennett and Galton (2004). Analogous to the frame problem, Shanahan (1995) describes a default reasoning problem that arises when an attempt is made to construct a logic-based calculus for reasoning about the movement of objects in a real-valued co-ordinate system. Shanahan's default reasoning (about spatial occupancy) approach is by far the most direct reference and application of a nonmonotonic approach within the spatial domain. As Shanahan (1995) elaborates:

> *"If we are to develop a formal theory of commonsense, we need a precisely defined language for talking about shape, spatial location and change. The theory will include axioms, expressed in that language, that capture domain-independent truths about shape, location and change, and will also incorporate a formal account of any nondeductive forms of commonsense inference that arise in reasoning about the spatial properties of objects and how they vary over time."*

Indeed, what Shanahan's all-encompassing theory refers to is a unification of spatial, temporal and causal concepts at ontological, representational and computational levels. Allen (1984) and Allen and Ferguson (1994) address much broader problem of developing a general representation of actions and events that uniformly supports a wide range of reasoning tasks, including planning, explanation, prediction, natural language understanding, and commonsense reasoning in general. According to Allen and Ferguson (1994, pg. 51), the novelty of their work is the combination of techniques (relevant to temporal reasoning and reasoning about action and change) into a unified framework that supports explicit reasoning about temporal relationships, actions, events and their effects. Bennett and Galton (2004) propose versatile event logic (VEL), which consists of a general temporal ontology and semantics encompassing many other representations such as the situation calculus and event calculus. The main motivation for the development of VEL is its use as a foundational representational framework for comparing and interfacing different AI languages.

Finally, some pointers on the outlook of this research and further researchable questions are in place. Since the underlying foundational theory is based on a customized version of the situation calculus formalism, existing situation calculus based high-level languages cannot be directly utilized. Furthermore, all existing languages support different functionalities [e.g., concurrency (Giacomo et al., 2000), real-time execution with incremental control (Grosskreutz & Lakemeyer, 2000)] and they also differ in terms of the theoretical underpinnings (pertaining to ramifications, treatment for causality, etc.). The outlook of the present research is primarily geared toward investigating implementation strategies for the proposed causal theory. Several theoretical elaborations are possible as well. Concurrency is an issue that has not been investigated in the specialised spatial reasoning domain. As such, support for concurrency is an important elaboration to the basic theory that needs detailed investigation. In this context, existing foundational work in this area [e.g., (Lin & Shoham, 1992), (Pinto, 1998)] or even situation calculus based high-level programming languages [e.g., conGolog (Giacomo et al., 2000), ccGolog (Grosskreutz & Lakemeyer, 2000)] that support concurrency are an interesting next step. On the application front, we envisage the development of a high-level spatial reasoning framework that supports easy *integration* of different qualitative spatial models (calculi) and possibly control mechanisms other than the situation calculus. For instance, it should be possible for a spatial agent to *reason* about dynamic topological and/or orientation models using either a situation calculus or event calculus-based control mechanism. It is envisaged that such a framework will facilitate easy integration of existing and independently developed spatial calculi, e.g., topology, orientation, distance, size, and control mechanisms, e.g., situation calculus, event calculus and fluent calculus.

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REFERENCES

Allen, J. F. (1984). Towards a general theory of action and time. *Artif. Intell., 23*(2), 123–154,

Allen, J. F., & Ferguson, G. (1994). Actions and events in interval temporal logic. *J. Log. Comput., 4*(5), 531–579.

Beller, A. (1991). Spatio/temporal events in a GIS. *Proceedings of GIS/LIS* (pp. 766–775).

Bennett, B., & Galton, A. P. (2004). A unifying semantics for time and events. *Artif. Intell., 153*(1–2), 13–48.

Claramunt, C., & Thériault, M. (1995). Managing time in GIS: An event-oriented approach. In *Proceedings of the International Workshop on Temporal Databases* (pp. 23–42). London, UK: Springer-Verlag.

Clarke, B. L. (1991). A calculus of individuals based on 'connection'. *Notre Dame Journal of Formal Logic, 22*(3), 204–218.

Cohn, A., & Hazarika, S. (2001a). Qualitative spatial representation and reasoning: An overview. *Fundam. Inf., 46*(1–2), 1–29.

Cohn, A., & Hazarika, S. (2001b). Qualitative self-localization using a spatio-temporal ontology: A preliminary report. In *IJCAI-2001 Workshop on Spatial and Temporal Reasoning with Agents' Focus*.

Davidson, D. (1967). Causal relations. *The Journal of Philosophy, 64*(21), 691–703.

Davidson, D. (1969). The individuation of events. In N. Rescher (Ed.), *Essays in honor of Carl G. Hempel* (pp. 216–234). Dordrecht: Reidel.

Freksa, C. (1991). Conceptual neighborhood and its role in temporal and spatial reasoning. In M. Singh & L. Travé-Massuyès (Eds.), *Decision*

Support Systems and Qualitative Reasoning (pp. 181–187). Amsterdam: North-Holland.

Freksa, C. (1992a). Using orientation information for qualitative spatial reasoning. In *Proceedings of the International Conference GIS—From Space to Territory: Theories and Methods of Spatio-Temporal Reasoning in Geographic Space* (pp. 162–178). London, UK: Springer-Verlag.

Freksa, C. (1992b). The double-cross calculus. <http://www.sfbtr8.uni-bremen.de/project/r3/QualitativeCalculi/Doublecross/Doublecross.html>.

Gerevini, A., & Renz, J. (2002). Combining topological and size information for spatial reasoning. *Artif. Intell., 137*(1–2), 1–42.

Giacomo, G., Lesperance, Y., & Levesque, H. J. (2000). Congolog, a concurrent programming language based on the situation calculus. *Artif. Intell., 121*(1–2), 109–169.

Giunchiglia, E., Lee, J., Lifschitz, V., McCain, N., & Turner, H. (2004). Nonmonotonic causal theories. *Artif. Intell., 153*(1–2), 49–104.

Gooday, J., & Cohn, A. (1996). Transition-based qualitative simulation. In *Proceeding of the 10th International Workshop on Qualitative Reasoning* (pp. 74–82). AAAI Press.

Gooday, J., & Galton, A. (1997). The transition calculus: A high-level formalism for reasoning about action and change. *J. Exp. Theor. Artif. Intell., 9*(1), 51–66.

Grosskreutz, H., & Lakemeyer, G. (2000). cc-golog: Towards more realistic logic-based robot controllers. In *Proceedings of the Seventeenth National Conference on Artificial Intelligence and Twelth Conference on Innovative Applications of Artificial Intelligence*, (pp. 476–482). AAAI Press/The MIT Press.

Hazarika, S. M., & Cohn, A. G. (2001). Qualitative spatio-temporal continuity. In *COSIT 2001: Proceedings of the International Conference on Spatial Information Theory* (pp. 92–107). London, UK: Springer-Verlag.

Kautz, H. A. (1986). The logic of persistence. In *Proceedings of the 5^{th} National Conference on Artificial Intelligence*, (pp. 401–405). Philadelphia, PA: Morgan Kaufmann.

Kim, J. (1971). Causes and events: Mackie on causation. *The Journal of Philosophy, 68*(14), 426–441.

Kim, J. (1976). Events as property exemplifications. In M. Brand & D. Walton (Eds.), *Action theory* (pp. 159–177). Dordrecht: Reidel.

Kowalski, R., & Sergot, M. (1986). A logic-based calculus of events. *New Gen. Comput., 4*(1), 67–95.

Lifschitz, V. (1994). Circumscription. In *Handbook of logic in artificial intelligence and logic programming (vol. 3)—Nonmonotonic reasoning and uncertain reasoning* (pp. 297–352). New York: Oxford University Press.

Ligozat, G., & Renz, J. (2004). What is a qualitative calculus? A general framework. In *Lecture Notes in Computer Science, Bibliographic Details*, (pp. 53–64). Germany: Springer-Verlag.

Lin, F. (1995). Embracing causality in specifying the indirect effects of actions. In *Proceedings of the 14th International Joint Conference on Artificial Intelligence (IJCAI-5)*, Montreal, Quebec, Canada (pp. 1985–1991). San Mateo, CA: Morgan Kaufmann.

Lin, F., & Reiter, R. (1994). State constraints revisited. *Journal of Logic and Computation, 4*, 655–678.

Lin, F., & Shoham, Y. (1992). Concurrent actions in the situation calculus. In *Proceedings of the 10th Annual Conference on Artificial Intelligence*, San Jose, CA, July 12–16, 1992 (pp. 590–595). AAAI Press/The MIT Press.

McCain, N., & Turner, H. (1997). Causal theories of action and change. In H. Shrobe & T. Senator (Eds.), *Proceedings of the Thirteenth National Conference on Artificial Intelligence and the Eighth Innovative Applications of Artificial Intelligence Conference* (pp. 460–465). Menlo Park, CA: AAAI Press.

McCarthy, J. (1977). Epistemological problems of artificial intelligence. In *Readings in Nonmonotonic Reasoning*, (pp. 46–52). San Francisco, CA: Morgan Kaufmann.

McCarthy, J., & Hayes, P. J. (1969). Some philosophical problems from the standpoint of artificial intelligence. In B. Meltzer & D. Michie (Eds.), *Machine intelligence 4* (pp. 463–502). Edinburgh University Press.

McIlraith, S. A. (2000). Integrating actions and state constraints: A closed-form solution to the ramification problem (sometimes). *Artif. Intell., 116*(1–2), 87–121.

Moratz, R., Renz, J., & Wolter, D. (2000). Qualitative spatial reasoning about line segments. In W. Horn (Ed.), *ECAI 2000, Proceedings of the 14th European Conference on Artificial Intelligence* (pp. 234–238).

Muller, P. (1998a). A qualitative theory of motion based on spatio-temporal primitives. In A. G. Cohn, L. K. Schubert, and S. C. Shapiro (Eds.), *Proceedings of the 6th International Conference on Principles of Knowledge Representation and Reasoning* (KR '98), Trenton, Italy, June 2–5, 1998 (pp. 131–143). Morgan Kaufmann.

Muller, P. (1998b). Space-time as a primitive for space and motion. *Formal Ontology in Information Systems (FOIS-98), 46*, 63–76.

Pianesi, F., & Varzi, A. (2000). Events and event talk: An introduction. In J. Higginbottom, F. Pianesi, & A. Varzi (Eds.), *In Speaking of events* (pp. 3–48). Oxford: Oxford University Press.

Pinto, J. (1998). Concurrent actions and interacting effects. In A. G. Cohn, L. Schubert, & S. C. Shapiro (Eds.), *KR: Principles of knowledge representation and reasoning* (pp. 292–303). Morgan Kaufmann.

Pinto, J. A. (1994). *Temporal reasoning in the situation calculus*. PhD thesis, Computer Science, University of Toronto, 1994.

Quine, W. (1960). *Word and object*. Cambridge, MA: MIT Press.

Randell, D. A., Cui, Z., & Cohn, A. (1992). A spatial logic based on regions and connection. In *KR'92. Principles of Knowledge Representation and*

Reasoning: Proceedings of the Third International Conference (pp. 165–176). San Mateo, CA: Morgan Kaufmann.

Reiter, R. (1991). The frame problem in situation the calculus: A simple solution (sometimes) and a completeness result for goal regression. In *Artificial Intelligence and Mathematical Theory of Computation: Papers in Honor of John McCarthy* (pp. 359–380). San Diego, CA: Academic Press Professional.

Reiter, R. (1993). Proving properties of states in the situation calculus. *Artif. Intell., 64*(2), 337–351.

Reiter, R. (2001). *Knowledge in action: Logical foundations for describing and implementing dynamical systems*. Cambridge, MA: MIT Press.

Sandewall, E. (1994). *Features and fluents (Vol. 1): The representation of knowledge about dynamical systems*. New York: Oxford University Press.

Shanahan, M. (1989). Prediction is deduction but explanation is abduction. In *Proceedings of the 11th International Joint Conference on Artificial Intelligence*, Detroit, MI, USA, August, 1989. Morgan Kaufmann.

Shanahan, M. (1993). Explanation in the situation calculus. In *Proceedings of the 13th International Joint Conference on Artificial Intelligence*, Chambéry, France, Aug. 28–Sept. 3, 1993. Morgan-Kaufmann.

Shanahan, M. (1995). Default reasoning about spatial occupancy. *Artif. Intell., 74*(1), 147–163.

Shanahan, M. (1997). *Solving the frame problem: A mathematical investigation of the common sense law of inertia*. Cambridge, MA: MIT Press.

Shoham, Y. (1988). *Reasoning about change: Time and causation from the standpoint of artificial intelligence*. Cambridge, MA: MIT Press.

Sosa, E., & Tooley, M. (1993). *Causation. Oxford readings in philosophy*. Oxford: Oxford University Press.

Thielscher, M. (1998). Introduction to the fluent calculus. *Electron. Trans. Artif. Intell., 2*, 179–192.

Worboys, M. F. (2005). Event-oriented approaches to geographic phenomena. *Int. Journal of Geographical Inf. Science, 19*(1), 1–28.