CS4 SE – Software Engineering

MODULARITY - PROCESSING ON LINKED LIST ORDERED

Our purpose is to analyze the functions needed to maintain and use a linked list whose nodes are ordered by some key, and, which is held on a random access file (i.e., a relative file, in COBOL parlance).

We will, for the beginning, ignore the way the list is stored. In a way it does not matter. See Fig. 1 below.

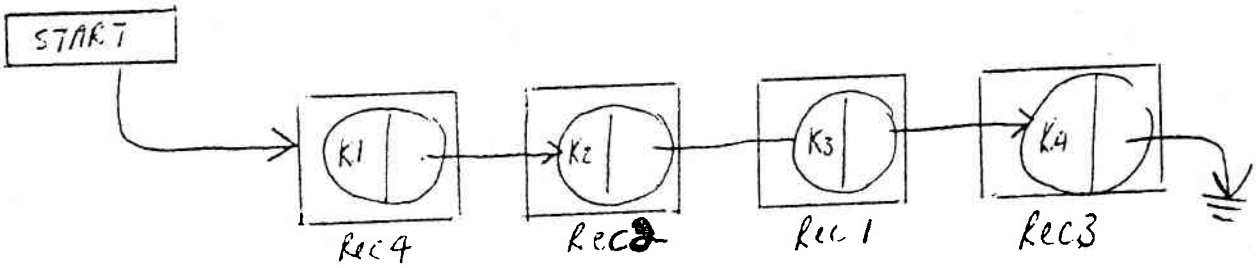


Figure I. A linked list on a random access file

Note that the list is to be ordered,

i.e. for all i , $1 \leq i < n$ where n (1)

is the number nodes in the list, it must always be true that

$$K_i \; \rho \; K_{i+1} \qquad (2)$$

where ρ is any binary relation which is a total order.

We begin by examining the operations to be performed on the list. They are tabulated below : (Table 1)

Table 1 - Operations On A Linked List

Name	Description
INSERT	puts a record in it's correct place
DELETE	removes a record
FIND	prints a record
CHANGE	alters either the key or data or both of a record
PRINTALL	prints the file in key order

Other functions might also be useful, e.g. :

CREATE	creates the file if it does not exist
DUMP	dumps the file in record order so that it can be examined visually.

However, we will look at those in Table I first.

TECHNIQUE

Examine each function "graphically" if appropriate, to see what it must do. DO NOT implement one function first.

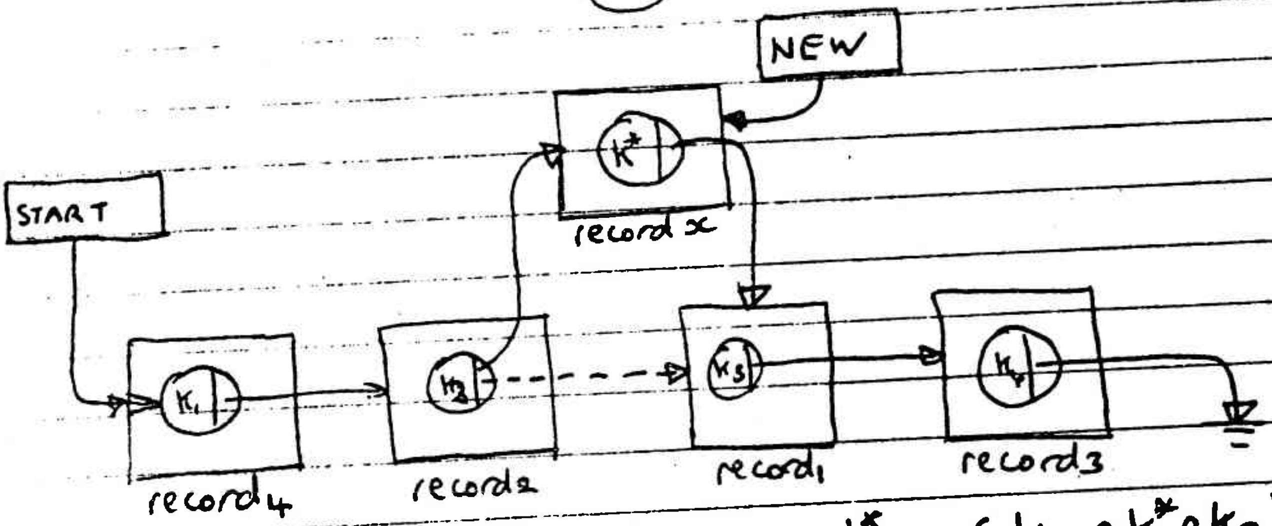
YOU MAY NOT PICK THE RIGHT ONE !

(You should look at the list in Table 1 and see if you can specify the right order of implementation).

In any case, we will "build" our solution from the bottom up so even picking the "right" function may not help !

The figures below show what will be done to change the list for the INSERT, DELETE and CHANGE OF KEY commands.

We assume that the node is held in the node pointed to by NEW



INSERT a node with key k^* ($k_2 \rho k^* \rho k_3$)

FIG 2. (Deleted links shown - - -)

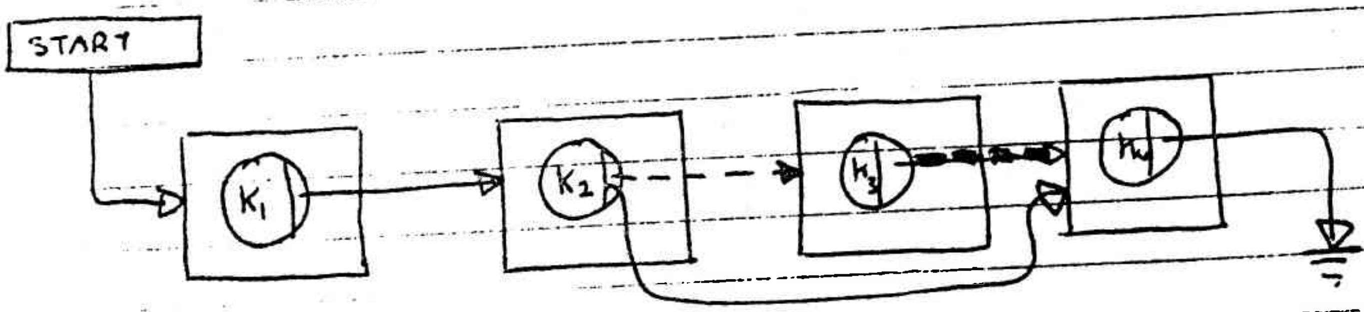


FIG 3. DELETE the node with key $k^* = k_3$

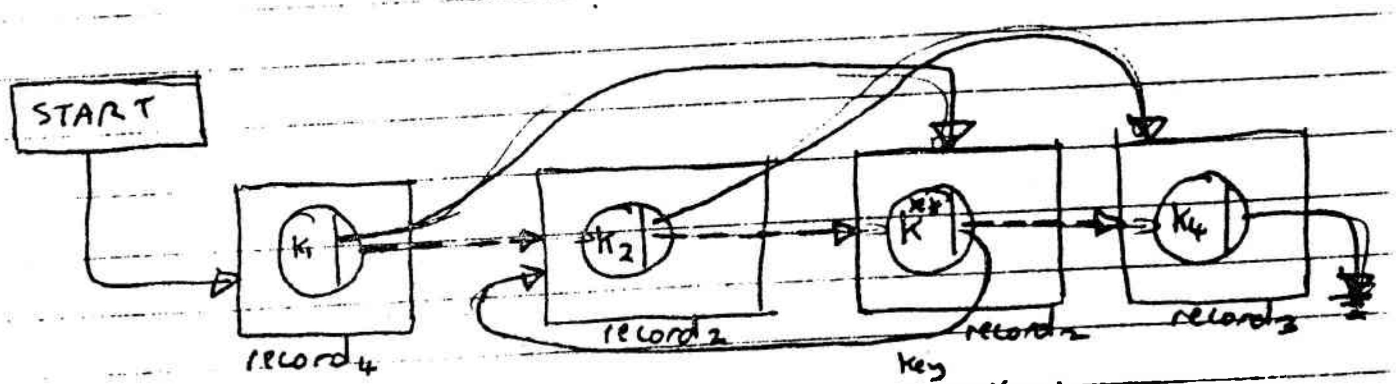


FIG 4. CHANGE the key of the node with key $k^* = k_3$ to k^{**} such that $k_1 \rho k_2^{**} \rho k_3$

We now have a clear picture of the way links will be changed, and can state them verbally.

However, let us refer to Table 1 and ask if we can extend it to include some simple statement of the error conditions :

In particular, we ought to note that :

Table 2

CONDITIONS FOR EACH FUNCTION

(a) We cannot insert K^* if there is no place§ for it in list (i.e. there must exist some K_i in the set of keys such that $K_i \rho K^* \rho k_{i+1}$)

(b) We cannot delete K^* if there is no key K_j in the list such that $K_i = K^*$.

(c) We cannot find K^* if there is no key K_i in the list such that $K_i = k^*$.

(d) We cannot change the key of a record from K^* to K^{**} unless :

(i) there exists $K_j = K^*$ in the list, and

(ii) there exists K_i such that $K_i \rho K^{**} \rho K_{i+}$ (i.e. there is a place for K_i).

§ Note that the concept of place (i.e. $\exists K_i : K_i \rho K^* \rho K_{i+1}$) is more general than that originally used, and allows for relations which are satisfied by equal keys.

2. "DISCOVERING" PRIMITIVE FUNCTIONS

2.1 LINKING

We can begin by examining the Figures 2 through 4 and note that :

(a) the insertion process involes :
two "linking" operations

(b) the deletion process involves :
one "linking" operation

(c) the change of key operation involves a delete followed by an insert; i.e. three linking operations !

SO, WE CONCLUDE THAT LINKING OPERATIONS ARE PRIMITIVES !

However, we do not know at this stage exactly what they look like.

2.2 FINDPLACE

Examining our list of CONDITIONS FOR EACH FUNCTION, we note that each is basically interested in the same question, but with a different answer, i.e.

either $K_i \ \rho K^* \ \rho K_{i+1}$ is to be true

or $K_i = K^*$ is to be true.

BOTH OF THESE INVOLVE A SEARCH OF THE LIST ! (OBVIOUSLY)
Hence we need to examine this search function (which is obviously a primitive) and see what it looks like.

Before we do, let us state each operation verbally :

INSERT K^*
Search for K^*.
If place found then
begin link new to K_{i+1}
end link K_i to new

DELETE K^*
Search for K^*, which will be at K_i if it exists
if found then link K_{i-1} to K_{i+1}

FIND K^*
Search for K^*
if found then print K^*

CHANGE K^* key to K^{**} (tricky)

Search for K^*, search for K^{**}

if K^* found and place for K^{**}

then begin

link K_{j-1} to K_{j+1} (remove K^* from chain)

link K_j to K_{i+1}

link K_i to K_j

change K_j key to K^{**}

end

(Note that CHANGE could be written :

DELETE K^*; copy data in K^* to a NEW

If successful then INSERT K^{**} (from NEW)).

However, we have missed an opportunity for optimization, since we are forced to begin our search for the keys from the beginning.

NOTE if $K^* \rho K^{**}$ holds, then the record with key K^* occurs before§ the K^{**}.

This suggests that :

THERE IS NO NEED TO RETURN TO THE START OF THE LIST,
WE CAN SEARCH FORWARD FROM THE POINT WHERE THE SEARCH FOR K^* TERMINATED !

We can only do this if the search function commences at a nominated starting point, not the beginning !

THIS SUGGESTS THAT THE SEARCH FUNCTION MUST BE TOLD WHERE TO START, i.e., that the starting point is a parameter.

We can now attempt to define the SEARCH function in more detail.

§ Unless ρ includes equality - in which case it does not matter.

SEARCH FUNCTION

SEARCH {start:in; search key:in; pointers to found node:out}

"pointers to found node" .... we should consider exactly what we mean by this, and how the search is to be carried out ......

Let us re-examine the list, and the search procedure.

CURRENT←START; if ~~NOT~~ (K^* ρ CURRENT↑. KEY) then finish.

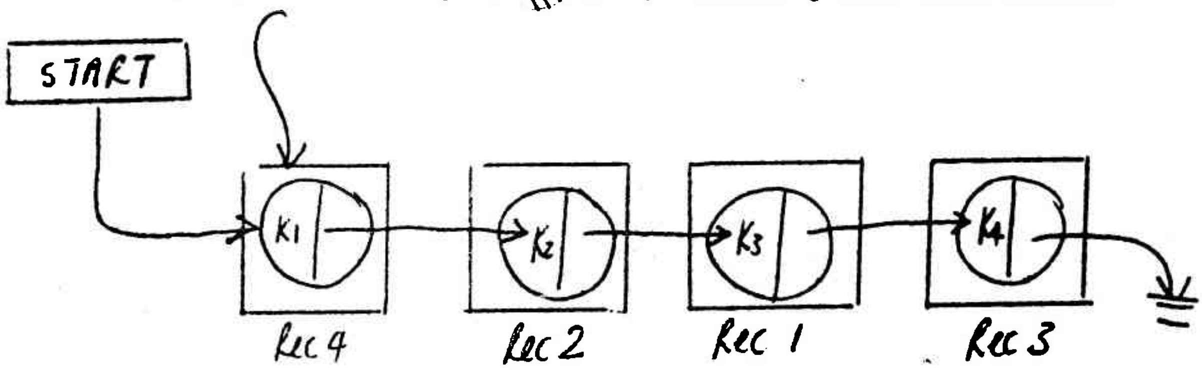


FIG. 5 - First step of Search

Notice only one step is considered !

Also the form of the relationship.

"~~NOT~~ (K^* ρ CURRENT↑. KEY)" the choice here depends upon the properties of the relation ρ , and the "ordering" if ρ includes equality.

(Consider the effect of CURRENT↑ KEY ρK^* → this must be negated)

However, referring to Figs. 2 through 4, we note that we require two pointers from SEARCH, formally, ~~yhodr yo yhr~~ there must be nodes which satisfy $K_i \rho K^* \rho K_{i+1}$, in all cases.

Hence, the search process should, on successive steps, look like Figs. 6 and 7.

CONSIDER ACSENDING KEYS,

AND ρ is $<$

i.e.	K1	K2	K3	K4
	3	5	7	9

If NEW.KEY = 8, it belongs between K3 & K4

If we start with CURRENT → START

If (K* ρ CURRENT↑.KEY) then stop

else step on one.

Check this for ρ is $>$ and

NEW.KEY = 10

START →	K1	K2	K3	K4
	9	7	5	3

If ρ is $\leq$ and $K^* = 7$

i.e.	K1	K2	K3	K4
	3	5	7	9

↑ Place for 7

A formal proof could be developed!

Search

while NOT (K = p currentA. key)

begin

if currentA. POINT ≠ null then

~~then~~ skip

end

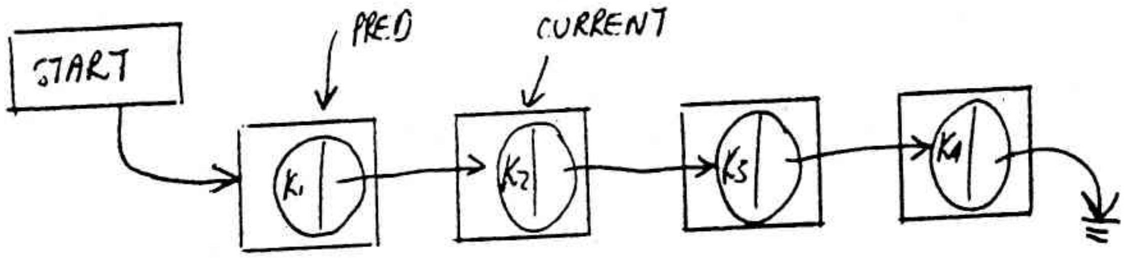


FIG.6 - Second Step

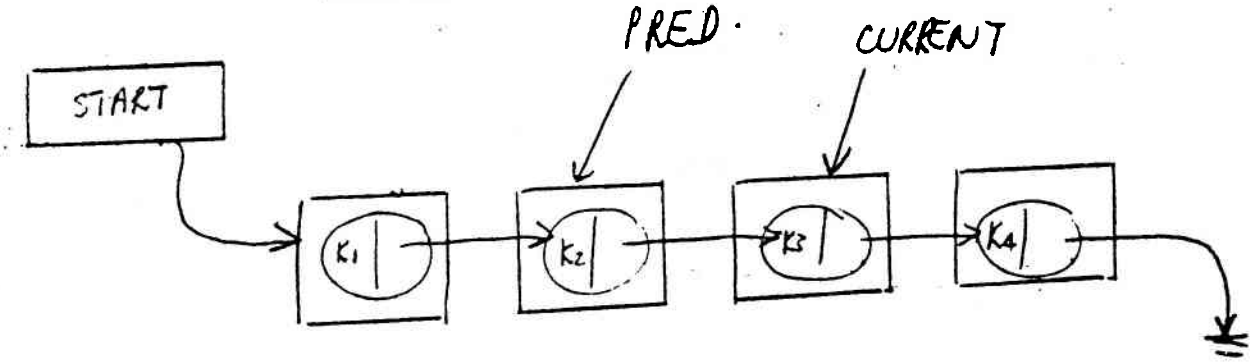


FIG.7 - Third Step

NOTE that the action between steps was

```
PRED←CURRENT                      | Fragment 1
CURRENT←CURRENT↑.PT               |
```

AND BEWARNED one should be aware that the operation CURRENT↑ may require a procedure call ! ! We will look at this last !

These two steps, then, combined with Fig. 5, read as :

```
if NOT(K* ρ CURRENT↑KEY) then PLACE FOUND
else
   begin
       PRED←CURRENT
       CURRENT←CURRENT.PT

   end
```

Fragment 2

The compound statement (between the begin and end) actually could be described as a "primitive" function STEP, e.g.

```
STEP {PRED, CURRENT:IN;PRED,CURRENT:OUT}
```

Fragment 3

Out temptation, at this point, for the INSERT, to just write

```
while NOT K* ρ CURRENT↑.KEY do
    begin
      PRED←CURRENT
      CURRENT←CURRENT↑.PT
    end
       (* place found *)
```

Fragment 4

Indeed, this would not be a bad choice.

We see that the search actually locates the place for an insertion.

We do not know the exact reason for the search termination

QUESTION

Do we have a useful primitive function ? Consider the requirements for an INSERT

ASSUME primitive SEARCH{K*,STARTPT:IN;PRED;CURRENT:OUT}

(* obtain first element, if necessary *)

(* initialize PRED, CURRENT *)

(* but first, check that list is not empty *)

Note : SPECIFICATION FOR SEARCH

CURRENT POINTS TO THE FIRST KEY FOR WHICH K* ρ CURRENT↑.KEY IS TRUE.

```
if START ≠ "null" then
  begin
      STARTPT←START
      PRED←START                                   Fragment 5
      CURRENT←START
      SEARCH{K*, STARTPT;PRED,CURRENT}
```

(* Assume that a place has been found - we have no warning at this point *)

(* Assume that the new node is pointed to by NEW *)

(* We now have the situation in Fig. 2 *)

(* Hence : - *)

```
      NEW↑.PT←PRED↑.PT     (* or NEW↑.PT←CURRENT*)
      PRED↑.PT←NEW                                   Fragment 6
  end
```

This is not really satisfactory. (Why ?)

It would be possible to see from Table 2 that we might have chosen a better primitive by examining the conditions which are involved.

Table 3 shows the "results" which are needed from the searches for each function.

TABLE 3 - SEARCH RESULTS

FUNCTION	RESULTS
INSERT	PLACE FOUND
DELETE	KEY FOUND
FIND	KEY FOUND KEY NOT FOUND
CHANGE KEY	KEY FOUND PLACE FOUND

It is clear then, that we must return a RESULT. However, we should ask ourselves how we handle the situation for insertions where key is to be inserted at the end of the list. This does of course, qualify as a "place found", but, how do we set the pointers and actually terminate the search ?

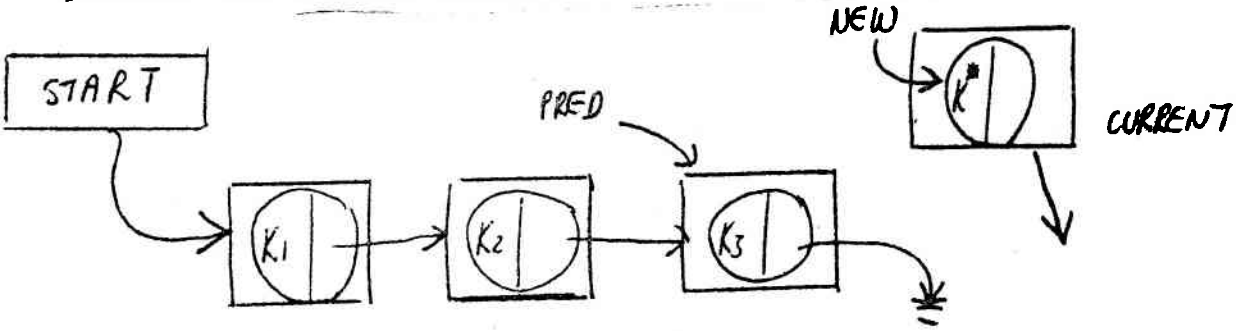


Fig. 8 K^* to go after K_3 - pointers at search termination

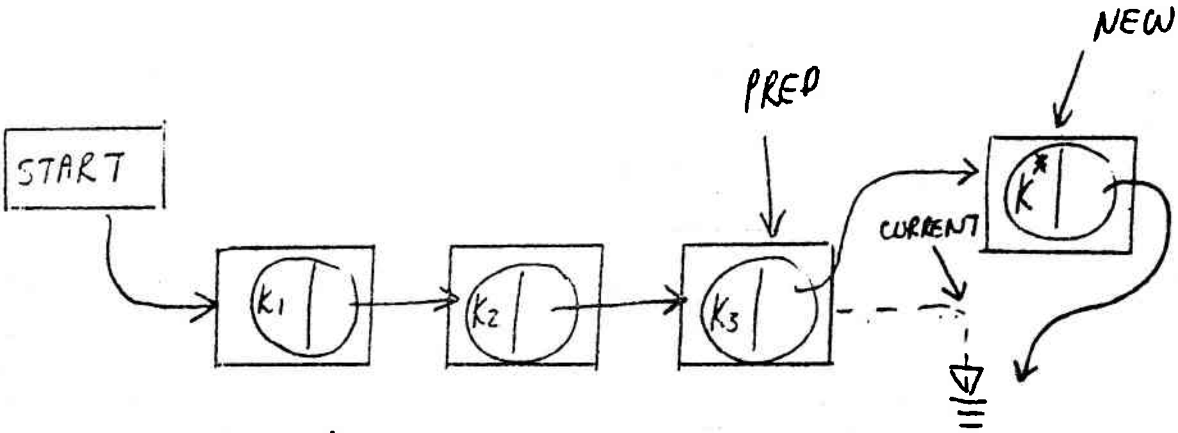


Fig. 9 K^* inserted after search

After the search, and the standard linking step

```
NEW↑.PT←PRED↑.PT
PRED↑.PT←NEW
```

SO, WE DO NOT NEED A SPECIAL RESULT FOR INSERT IF THERE IS NO "TRUE" PLACE IN THE LIST !

However, we note that we do need a result "key not found" for FIND (see Table 3a).

Before considering this, let us consider the "standard linking step", and see what happens if we need to INSERT before K1.

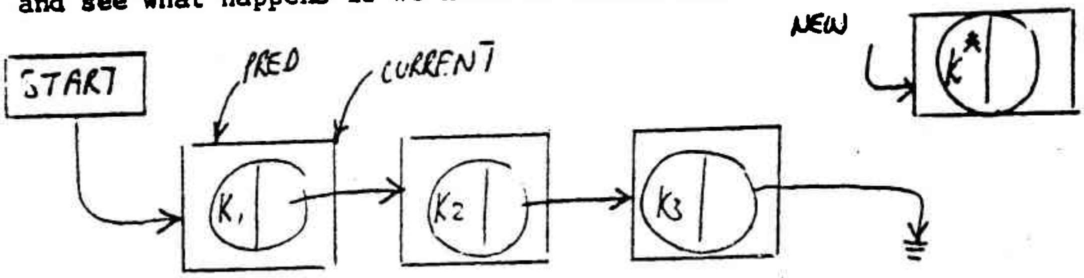


Fig.10 K^* to go before K_1 - pointers at search termination

The standard linking operation will not work in this case, since it assumes that PRED and CURRENT are distinct, which they are not.

Note that it is START which is to be altered, see Fig. 11.

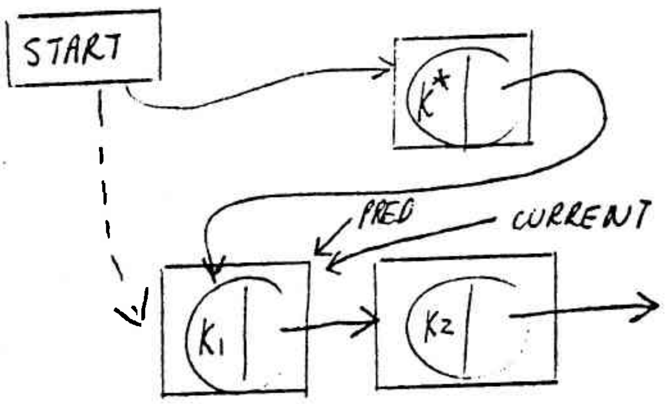


Fig.11 - After Correct Linking

The standard linking operation, as performed will not work because it assumes that PRED points to a node.

A possible solution

Let start point to start node. (See Fig. 12)

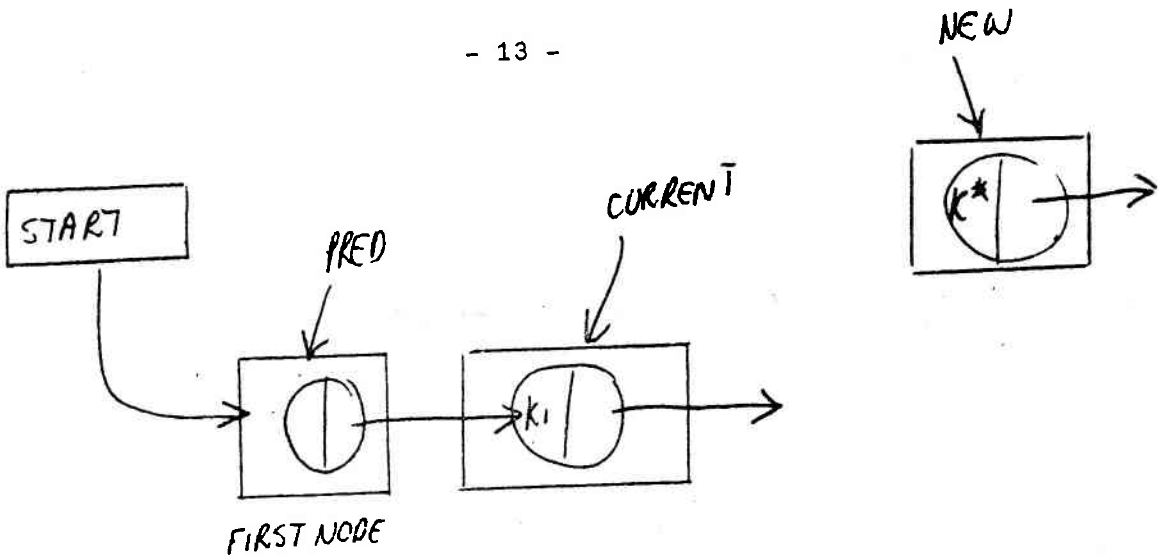


Fig. 12 Use of a Header Node - end of search

Inserting via the standard linking sequence puts NEW after the first node, and works well.

YOU MAY OF COURSE DO THIS IF PRACTICAL

However, one may not be able to do this because

(a) the first node may not be identical to other nodes.

(b) the nodes may be physically large, and therefore it may be impractical to hold more than a few in memory.

(Objection a) can be overcome by the use of undiscriminated unions in PASCAL and REDEFINES in COBOL - but care is needed !)

so let us revert to Fig. 10, i.e. START points to the first real link.

A Linguistic Interlude

It is clear that a procedure

```
LINKIN(NEWPT,PREDPT,NEW)
     begin
          NEWPT←PREDPT
          PREDPT←NEW
     end
```

Fragment 6

will work if called by

```
LINKIN(NEW↑.PT,PRED↑.PT,NEW)
```

CONSIDER the language statement

"PRED↑.PT IS START"

Semantics

An assignment to PRED↑.PT alters START unless PRED has been altered since the execution of the IS statement.

We could then write

```
PRED↑.PT IS START;
CURRENT←START

SEARCH{K*, START;PRED,CURRENT}

LINKIN{NEW .PT,PRED↑.PT,NEW}
```

Fragment 7

end of interlude

However, we cannot.
Hence we must write, for our insert :

```
PRED←START
CURRENT←START

SEARCH{K*,STARTPT;PRED,CURRENT}

IF PRED = START THEN
        LINKIN{NEW↑.PT;START;NEW}
        ELSE
        LINKIN{NEW↑.PT,PRED↑.PT,NEW}
```

Fragment 8

which is not as bad as all that !

Notice that we have not worried about the problem of equality in the search.

LET US NOW EXAMINE THE OTHER FUNCTIONS

DELETE could be described as :

search for key
link it out.

This translates to :

```
PRED←START
CURRENT←START

SEARCH{K*,STARTPT;PRED,CURRENT}

IF  key is found THEN (*  PRED points to KEY *)      Fragment 9

BEGIN
   IF PRED=START THEN
      START←PRED↑.PT, return(PRED)
      ELSE
```

FIND

Find is basically a delete with a different action.

```
PRED←START
CURRENT←START

SEARCH{K*,STARTPT;PRED,CURRENT}                      Fragment 10

IF key is found THEN (* PRED points to Key *)
   DISPLAY (KEY)
```

CHANGE Key value K^{*} to K^{**}

here we would code :

IF K^{*} ρK^{**} in then

case 1,

else (* K^{**} ρK^{*}) case 2.

where case 1 is a "procedure" which does a change in the first case, and K^{**} ρK^{*} does it in the second case.

SO suppose we invent a procedure "CHANGE-IN-ORDER" with two parameters, K_1 and K_2 but, this becomes a real mess ! (try it and see).

(Back out a little !)

We need to ask ourselves -
"What do we need to perform this function ?"

From Fig. 4, we see we need four pointers.

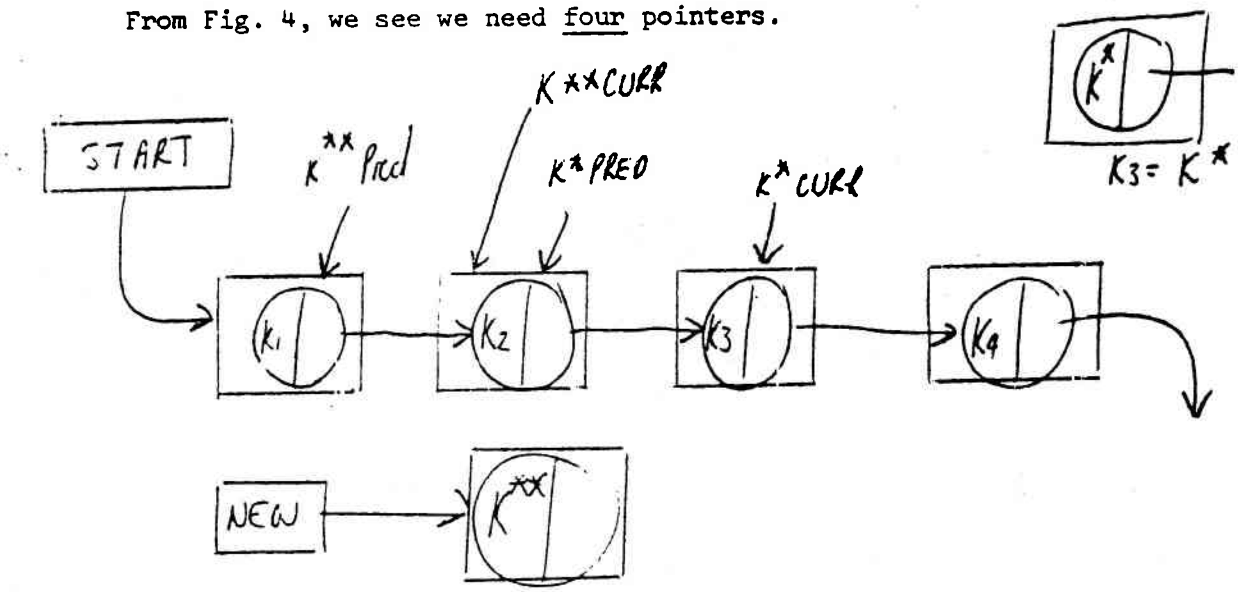


Fig. 13 Pointers for Key Change

These four pointers can be picked up by two calls to search. The order of these searches depends upon the order of the two keys.

```
IF K* ρ K** THEN
 BEGIN
   K*PRED←START;
   K*CURR←START;
   SEARCH(K*,STARTPT;K*PRED,K*CURR)
   K*RES=RES
   (* NOW FIND THE OTHER PAIR *)
   K**PRED←K*CUR
   K**CURR←K*CUR
   (* start search from the point just reached *)
                                                        Fragment 11
   SEARCH(K* STARTPT;K*PRED,K*CURR)
 END ***RES=RES
ELSE (** ρ K*) (* K** ρ K* *)
 BEGIN
   (* repeat above using K** in place of K*, and vice versa *)
 END
   IF K*RES=FOUND and K**RES=PLACE FOUND THEN
   BEGIN
OKDELETE  OK DELETE(K*PRED;K*CURR,START)
   alter Key of K*CURR to K**
   OKLINKIN (K*CURR↑.PT, K**PRED↑.PT,K*CURR,START)
   END
     (* process errors *).
```

Note that we have used the section of code from Fragment 8 of beginning "IF PRED= " as the procedure OKLINKIN.

THERE ARE A NUMBER OF THINGS WHICH STILL NEED TO BE CLEANED UP.

THESE INCLUDE :

(A) <u>clumsy use of procedure SEARCH and its parameters</u>

SEARCH ought to begin from STARTPT - this would save some initialization.

(B) The result of SEARCH is not a parameter, nor has it been defined. That is Ok, at least we know what results we require - or do we ?

(C) There is not test for end of list.
otherwise we are in good shape.

LET US EXAMINE THE RESULTS

Consider ρ is ≤ .

Then

```
SEARCH{KEYSOUGHT,IN;STARTPT,IN:PRED,CURR:OUT}
CURR←STARTPT

WHILE NOT CURR=null DO
IF KEYSOUGHT  ≤  CURR .KEY THEN

   GO TO FOUNDPLACE
  ELSE BEGIN
         PRED←CURR
         CURR←CURR↑.PT
       END
       FOUNDRES←NOTFOUND;
       RETURN  (* exits procedure *)

FOUNDPLACE :  IF KEYSOUGHT=CURR↑.KEY THEN
                       FOUNDRES←FOUND
              ELSE     FOUNDRES←NOTFOUND
```

Fragment 12

we note that there is always a place for the key, in this case - found or not.

Exactly what we need depends upon the relationship ρ .

ρ may ~~or may not~~ include equality (e.g. ρ = ≤) or it may not, (e.g. ρ = >). Clearly, if we stop our search when we have found the first item for which ρ is true, then the key of the "sought" item may or not be equal to that of the stopping point.

<u>It is interesting to note that</u>,

while our external action does not depend on the relation (we are interested in three results, key found, key not found, place found). The action <u>inside</u> SEARCH does indeed.

Consider ρ is ≤

then

```
SEARCH{KEYSOUGHT,IN:STARTPT,IN:PRED,CURR:OUT;}
CURR←STARTPT

WHILE NOT CURR=null DO
IF KEYSOUGHT ≤ CURR↑.KEY THEN
   GO TO FOUNDPLACE
 ELSE BEGIN
        PRED←CURR
        CURR←CURR↑.PT
      END
      FOUNDRES←NOTFOUND;
      RETURN (* exits procedure *)
FOUNDPLACE: IF KEYSOUGHT=CURR↑.KEY THEN
                    FOUNDRES←FOUND
            ELSE    FOUNDRES←NOTFOUND
```

Fragment 12

We note that there is always a place for the key, in this case - found or not.

Consider ρ is <

then

```
SEARCH{KEYSOUGHT,STARTPT:IN;PRED,CURR;OUT}
CURR←STARTPT
WHILE NOT CURR=null DO
IF KEYSOUGHT<CURR↑.KEY THEN GO TO FOUNDPLACE
IF KEYSOUGHT=CURR↑.KEY THEN GO TO FOUND
ELSE BEGIN
        PRED←CURR
        CURR←CURR↑.PT
      END
FOUNDPLACE: FOUNDRES←FOUNDPLACE RETURN;
FOUND: FOUNDRES←FOUND  RETURN;
```

Fragment 13

Comparing FRAGMENTS 12 and 13 we see that they are equivalent. (Why ? make sure you see why !)

Except that we are calling the result of FRAGMENT 12 "NOTFOUND" instead of "PLACE FOUND"

THIS WILL NOT ALWAYS BE TRUE, SO, THE DETAIL OF SEARCH WILL NEED TO BE RE-WRITTEN FOR EACH CASE.

NOTE this sort of problem can be easily handled when a procedure can be passed as a parameter (HOW ?)

What is important, however, is that we ~~conceude~~ conclude that if the list is ordered by "ρ" then, when SEARCH STOPS

(a) the target may be found

(b) if it is not found we have the place for an insertion.

SO, WE ARE ONLY INTERESTED IN TWO RESULTS, NOT THREE. EXCEPT THAT FOUND MAY OR MAY NOT MEAN PLACEFOUND!

NOTE ALSO FROM FRAGMENT 12, we have cleared up the problem of the start and initialization of PRED,CURR.

We assume the SEARCH commences from STARTPT .... lets clean it up finally !

SEARCH

Definition

Searchs for the list item with key KEYSOUGHT commencing from the node pointed to by STARTPT.

It stops when either :

(a) the KEYSOUGHT is found

or (b) it's place is found

and returns separate indications for these two.

Note that when SEARCH stops with ~~CVPR~~ CURR = STARTPT, PRED is meaningless,

otherwise PRED points to the successor to CURR↑, and CURR points to the first item for which ρ is true.

CODE FOR SEARCH

```
procedure   SEARCH{KEYSOUGHT,STARTPT:IN;PRED,CURR,RESULT:OUT}
```

```
CURR←STARTPT

  WHILE NOT CURR=NULL DO{search while}
    IF KEYSOUGHT ρ CURR↑.KEY THEN {have we a termination}
          BEGIN{check for equality}
             IF KEYSOUGHT=CURR↑.KEY
                          THEN RESULT←FOUND
                               RETURN
                          ELSE GO TO FOUNDPLACE
          END {check for equality}
    ELSE {have we a termination? no, not here}
       BEGIN {step forwards one link}
             PRED←CURR
             CURR←CURR↑.PT
       END {step forwards one link}
    {ENDIF have we a termination, no, we will go on}
    {ENDWHILE search while}

  FOUNDPLACE: {we have found a place, either by termination
               or by finding a place}

              RESULT←PLACEFOUND
              RETURN
  END {SEARCH}.
```

<u>NOW WE CAN CODE OUR PROCEDURES.</u>

<u>START WITH FIND</u>

```
PROCEDURE FIND(START,KEYSOUGHT:IN)
  SEARCH(KEYSOUGHT,START,PRED,CURR,RESULT)
  IF RESULT=FOUND THEN PRINT(CURR)
                  ELSE LOGERR("KEYSOUGHT")
  END
```

NEXT DELETE

```
PROCEDURE DELETE(START,KEYSOUGHT;IN)
    SEARCH(START,KEYSOUGHT,PRED,CURR,RESULT)
    IF RESULT=FOUND THEN
    BEGIN {PROCESS THE found record}
        IF CURR=START {Bypass first item}
           THEN LINK(START,CURR↑.PT)
           ELSE {all other cases}
             LINK(PRED↑.PT,CURR↑.PT}
           {end of nested if}
        RECLAIM(CURR) {put object pointed to by
                       CURR on delete chain}
     END
     ELSE   LOGERR("RECORD NOT FOUND"

 PROCEDURE INSERT(START,KEYSOUGHT,BEGIN,RESULTTAB)
  {First, find a place for insertion}
   SEARCH(START,KEYSOUGHT,PRED,CURR,RESULT)
     IF  RESULTAB["SEARCH",RESULT] = FOUNDPLACE
       THEN
       BEGIN {perform insertion} GET_FREE_REC(NEW)
         IF CURR=START {Bypass first item}
            THEN LINKIN(NEW↑.PT,START,CURR↑.PT)
            ELSE LINKIN(NEW↑.PT,PRED↑.PT,CURR↑.PT)
       END
      ELSE   LOGERR("NO PLACE FOR KEY")
   END {procedure complete}
```

Finally, the most complicated of all, we re-write Fragment 11.

```
PROCEDURE CHANGE_IN_ORDER(START,OLDKEY,NEWKEY,RESTAB:IN:NEREL;INOUT);

  BEGIN
    IF OLDKEY  ρ  NEWKEY THEN
    BEGIN
      SEARCH(START,OLDKEY,OLDPRED,OLDCURR,OLDRES)
      IF RESTAB[SEARCH,OLDRES] ≠ FOUND THEN LOGERR("OLD KEY NOT FOUND");
      SEARCH(OLDCURR,NEWKEY,NEWPRED,NEWPRED,NEWRES)
        {Note we continue from the original found point}

      IF RESTAB[SEARCH,NEWRES] ≠ PLACEFOUND
                                    THEN LOGERR("NEWKEY HAS NO PLACE")

    END {reverse case}

      SEARCH(START,NEWKEY,NEWPRED,NEWCURR,NEWRES)
        IF RESTAB[SEARCH,NEWRES] ≠ FOUNDPLACE
                              THEN LOGERR ("NOPLACE FOR NEWKEY")
      SEARCH(NEWPRED,OLDKEY,OLDPRED,OLDCURR,OLDRES)
        IF RESTAB[SEARCH,OLDRES] ≠ FOUND
                                    THEN
```

FRAGMENT 14

I must then return to the previous definitions and simplify them (DO this, re-write LINKIN as well).

SECONDLY I ask myself a question -
What am I trying to do ?

ACTUALLY I want to search for the "least" key, then the other one. THEN I want to make the necessary changes !

SO If I can somehow "tag" the keys so that I
(a) search for the least key first, the other key next
and (b) remember which key was which, I will succeed.

HOWEVER, I do have a technique for doing the reverse.
I can set up a key to be the lowest key, and remember whether it is the NEW Key or the OLD Key, and vice versa.
The routine "CHANGE_IN_ORDER" follows :

```
PROCEDURE CHANGE_IN_ORDER(START,OLDKEY,NEWKEY,RESTAB:IN:NEWREL:INOUT)
  BEGIN  {SET UP KEYS for correct order of search}
         {note we simulate associative memory}

  IF OLDKEY  ρ  NEWKEY
         THEN OLDPT←FIRST:MEWPT←SECOND
         ELSE OLDPT←SECOND:NEWPT←FIRST

         {"remember" keys}

         KEY[OLDPT] = OLDKEY;
         KEY[NEWPT] = NEWKEY;

         SEARCH(START,KEY[FIRST],PRED[FIRST],CURR[FIRST],RES[FIRST])
         SEARCH(PRED[FRST],KEY[SECOND],PRED[SECOND],CURR[SECOND],
                                                  RES[SECOND])

 NOTICE THIS - WHAT HAVE I DONE
         LOGCHANGEI R(RESTAB RES,OLDPT,ERRME S,ERRFLAC )
         LOGCHANGEI R(RESTAB RES,OLDPT,ERRME S,ERRFLAC )
         IF NOT ERI FLAC THEN BEGIN
                {FIRST, delete the OLD KEY's record}
              LINK(START,PRED[OLD],CURR[OLD])
                {alter the keys}
                CURR[OLD]↑.KEY = KEY[NEWPT]
              LINKIN(START,CURR[OLD],PRED[NEW],CURR[NEW])
           END
         END {of change of key} .

     Note 14 lines of executable code.
```

KR/MG