

Technology adoption — Total Cost assessment

The failure of CASE (and other tools) is often ~~be~~ considered due to poor management. There are other possible causes — consider the costs of adopting a new technology: —

- C_{TP} — Cost of the technology — products — i.e. S/W, H/W, manuals etc.
- C_{TT_s} — Cost of ^{short-term} training — courses etc that are required to bring people to an appropriate level of competence — external costs.
- C_{TT_L} — Cost of long-term training — this can show up as reduced productivity, or, internal training.

The first two items are readily measurable — the second is hard.

Another way of looking at this is through the lost-productivity resulting from the use of a new technology — this can be considered to be a traditional learning curve model — i.e. exponential.

Let P_0 be the productivity in
~~output/time (e.g. LOC/person-month)~~
 output/person-time (e.g.) LOC/person-month

Let P_n be the productivity after adopting
 the new technology.

We assume P_0 is constant. $= P_0$

We assume $P_n(\tau) = A(1 - e^{-k\tau})$ ①

that is — the productivity after adopting the new
 technology increases with time τ

To obtain a value for A — — —

From ①, when $\tau \rightarrow \infty$, $P_n(\tau)$ is
 effectively constant since $e^{-k\tau} \rightarrow 0$

$\therefore \lim_{\tau \rightarrow \infty} P_n(\tau) = A = P_n$, the steady-state
 productivity. ②

$\therefore \boxed{P_n(\tau) = P_n(1 - e^{-k\tau})}$ ③

The output O can be calculated as

$$O = \int_0^{\tau} P(\tau) d\tau \quad ④$$

i.e. the integral of the instantaneous product of productivity and an increase in time.

The increase in output is at

$$\text{Simply } O_n - O_0$$

↳ new output ↳ original output

being more formal, after some time τ ,

$$\Delta O(\tau) = O_n(\tau) - O_0(\tau) \quad (5)$$

(Alternatively -- the increase in output is the integral over time of the increase in productivity -- the result is the same)

$$\Delta O(\tau) = \int_0^{\tau} p_n(\tau) d\tau - \int_0^{\tau} p_0(\tau) d\tau \quad \text{equation 4} \quad (6)$$

$$= \int_0^{\tau} (p_n(\tau) - p_0(\tau)) d\tau \quad (7)$$

Substitution of $p_n(\tau)$ and $p_0(\tau)$ ← equation 3

$$\Delta O(\tau) = \int_0^{\tau} [p_n(1 - e^{-k\tau}) - p_0] d\tau \quad (8)$$

$$\Delta D(\tau) = \left[P_n \left(\tau + \frac{1}{k} e^{-k\tau} \right) - P_0 \tau \right] \tau$$

$$\Delta D(\tau) = \frac{P_0}{P_0} \left[\frac{P_n}{P_0} \left(\tau + \frac{1}{k} e^{-k\tau} \right) - \tau \right] \tau$$

Converting the above we obtain

$$\Delta D(\tau) = P_0 \left\{ \left[\frac{P_n}{P_0} \left(\tau + \frac{1}{k} e^{-k\tau} \right) - \tau \right] - \frac{P_n}{P_0} \cdot \frac{1}{k} \right\} \quad (11)$$

$$= P_0 \left[\frac{P_n}{P_0} \tau \right]$$

$$\Delta D(\tau) = P_0 \left[\tau \left(\frac{P_n}{P_0} - 1 \right) - \frac{P_n}{P_0 k} \left(1 - e^{-k\tau} \right) \right] \quad (12)$$

We can approach (12) as follows to obtain a useful value of k .

Reverting to equation (3)

$$P_n(\tau) = P_n (1 - e^{-k\tau})$$

We can obtain a value for k in terms of some useful project parameter e.g., the value of τ (τ_c) at which the productivity is 95% of its maximum. i.e. τ_c is the point in time at which learning is complete.

$$\begin{aligned} \therefore P_n(\tau_c) &= 0.95 P_n \\ \therefore P_n(1 - e^{-k\tau_c}) &= 0.95 P_n \\ \therefore e^{-k\tau_c} &= 0.05 \\ \therefore e^{+k\tau_c} &= 20 \\ \therefore k &= \frac{\ln 20}{\tau_c} \end{aligned}$$

(14)

$$\ln 20 \approx 2.9957 \approx 3$$

$$\therefore k = \frac{3}{\tau_c}$$

(15)

Substituting in (12) we obtain:

$$\begin{aligned} \Delta O(\tau) &= P_0 \left[\tau \left(\frac{P_n}{P_0} - 1 \right) - \frac{P_n}{3P_0} \left(1 - e^{-\frac{3\tau}{\tau_c}} \right) \right] \\ &= P_0 \left[\tau \left(\frac{P_n}{P_0} - 1 \right) - \frac{\tau_c}{3} \cdot \frac{P_n}{P_0} \left(1 - e^{-\frac{3\tau}{\tau_c}} \right) \right] \end{aligned}$$

(16)

If we extract τ_c — we have

$$\Delta O(\tau) = P_0 \tau_c \left[\frac{\tau}{\tau_c} \left(\frac{P_n}{P_0} - 1 \right) - \frac{P_n}{3P_0} \left(1 - e^{-\frac{3\tau}{\tau_c}} \right) \right]$$

(17)

$\frac{P_n}{P_0}$ = is the productivity improvement ratio R_p

$P_0 \tau_c$ = original output that would have been completed with no improvement — after learning is complete

We can call $P_0 \tau_c = O_{0\tau_c}$

$$\therefore \Delta O(\tau) = O_{0\tau_c} \left[\frac{\tau}{\tau_c} \left(\frac{P_n}{P_0} - 1 \right) - \frac{P_n}{3P_0} \left(1 - e^{-3\frac{\tau}{\tau_c}} \right) \right] \quad (18)$$

n.b. $\frac{\tau}{\tau_c}$ is the normalized time in terms of ~~the~~ τ_c the learning completion time,

∴ Behaviour based on best - worst case of "learning".

The best possible result is learning is completed before the adaption occurs — i.e. $\tau_c = 0$

* The worst possible result is that learning is never complete

i.e. ~~$\tau_c \rightarrow \infty$~~ $\tau_c > \tau$

From equation (16) $\left\{ \Delta O(\tau) = P_0 \left[\tau \left(\frac{P_n}{P_0} - 1 \right) - \frac{\tau_c}{3} \frac{P_n}{P_0} \left(1 - e^{-3\frac{\tau}{\tau_c}} \right) \right] \right\}$

$$\Delta O(\tau) \Big|_{\tau_c=0} = P_0 \tau \left(\frac{P_n}{P_0} - 1 \right) \quad (19)$$

~~$$\Delta O(\tau) \Big|_{\tau_c \rightarrow \infty} = P_0 \left[\tau \left(\frac{P_n}{P_0} - 1 \right) - \frac{\tau_c P_n}{3 P_0} \right] \quad (20)$$~~

actually, τ_c

In the first case, ($\tau_c = 0$) use equation

16
TAD
7

$$\text{i.e. } \Delta O(\tau) = P_0 \left[\tau \left(\frac{P_n}{P_0} - 1 \right) - \frac{\tau_c P_n}{3 P_0} \left(1 - e^{-\frac{3\tau}{\tau_c}} \right) \right]$$

$$\tau_c \rightarrow 0 \quad \Delta O(\tau) \rightarrow P_0 \tau \left(\frac{P_n}{P_0} - 1 \right)$$

(19)

~~Second case ($\tau_c \rightarrow \infty$) use equation~~

~~17~~

~~$$\text{i.e. } \Delta O(\tau) = P_0 \tau_c \left[\frac{\tau}{\tau_c} \left(\frac{P_n}{P_0} - 1 \right) - \frac{P_n}{3 P_0} \left(1 - e^{-\frac{3\tau}{\tau_c}} \right) \right]$$~~

~~(20)~~

~~when $\tau_c \rightarrow \infty$ (i.e. becomes very large)~~

~~$$\Delta O(\tau) \rightarrow 0 \quad \text{since}$$~~

~~$$1 - e^{-\frac{3\tau}{\tau_c}} \rightarrow 1 - e^{-\frac{3\tau}{\infty}} \rightarrow 1 - e^{-0} \rightarrow 0$$~~

~~(in practice - the worst case is simply~~

~~Project completion time $\tau \ll \tau_c$, so $\tau_c \rightarrow \infty$~~

~~does not have much meaning~~

Looking at equation (16), we find some difficulty in interpreting what happens to the

term $\frac{\tau_c P_n}{3 P_0} \left(1 - e^{-\frac{3\tau}{\tau_c}} \right)$ when $\tau_c \rightarrow \infty$

when $\tau_c \rightarrow \infty$, $\frac{3\pi}{\tau_c} \rightarrow 0$

$$\therefore e^{-\frac{3\pi}{\tau_c}} \rightarrow e^{-0} \rightarrow 1$$

$$\therefore (1 - e^{-\frac{3\pi}{\tau_c}}) \rightarrow 0$$

(20)

(21)

but $\frac{\tau_c}{3} \frac{P_n}{P_0} \rightarrow \infty$ i.e. we have

$\infty \cdot 0$ the meaning of this is not clear —

However we are discussing τ_c v. large really, not $\tau_c \rightarrow \infty$ i.e. $\frac{3\pi}{\tau_c}$ v. small

Hence we should look at

$$e^{-x}, \quad x \text{ v. small } (x \ll 1)$$

We can use the power series for e^x

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!}$$

(22)

$$\therefore e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!}$$

(23)

If $x = \frac{3\pi}{\tau_c}$ and $\tau_c \gg \pi$

then $x \ll 1$ hence

$$e^{-x} \approx 1 - x$$

(24)

if $\frac{3T}{\tau_c} = \frac{1}{10}$ $e^{-x} = 1 - \frac{1}{10} + \frac{1}{2 \cdot 100} - \frac{1}{6 \cdot 1000}$

if $\frac{3T}{\tau_c} = \frac{1}{100}$ $e^{-x} = 1 - \frac{1}{100} + \frac{1}{2 \cdot 10000} - \dots$

So we ~~approx~~ substitute $e^{-\frac{3T}{\tau_c}} = 1 - \frac{3T}{\tau_c}$ (16)

$$\begin{aligned} \therefore \Delta O(\tau) \Big|_{\tau_c \rightarrow \infty} &= P_0 \left[\tau \left(\frac{P_n}{P_0} - 1 \right) - \frac{\tau_c}{3} \frac{P_n}{P_0} \left(1 - \left(1 - \frac{3T}{\tau_c} \right) \right) \right] \\ &= P_0 \left[\tau \left(\frac{P_n}{P_0} - 1 \right) - \uparrow \frac{P_n}{P_0} \right] \end{aligned} \quad (26)$$

$$= -P_0 T \quad (27)$$

(this is the hard way - consider zeros)

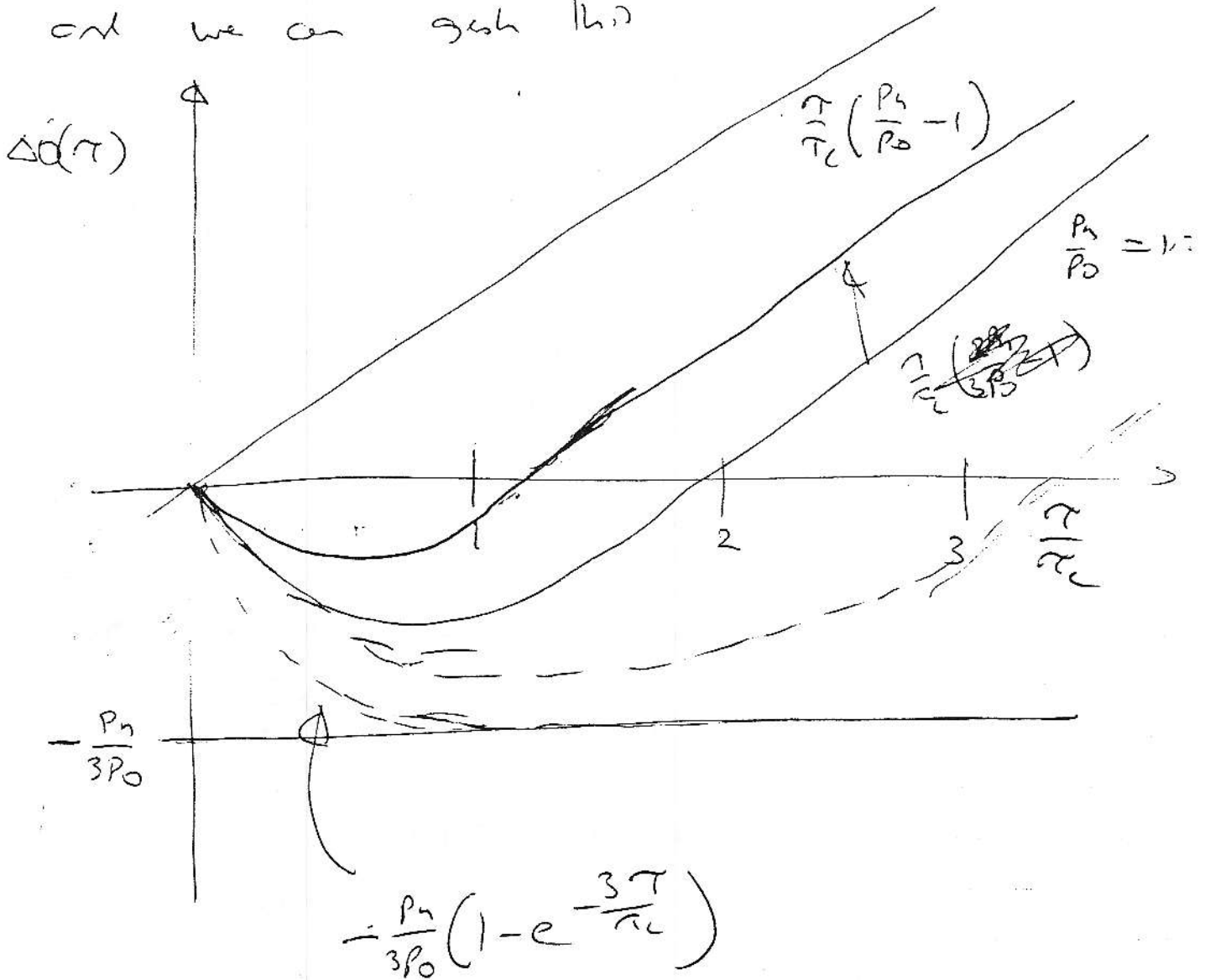
back to $P_n(\tau) = P_n(1 - e^{-\frac{3T\tau}{\tau_c}})$

$$\tau_c \rightarrow \infty \quad \lim_{\tau_c \rightarrow \infty} \tau e^{-P_n(\tau)} = 0$$

Consider equation (17)

$$\Delta O(\tau) = P_0 \tau_c \left[\frac{\tau}{\tau_c} \left(\frac{P_n}{P_0} - 1 \right) - \frac{P_n}{3P_0} \left(1 - e^{-\frac{3\tau}{\tau_c}} \right) \right]$$

and we can graph this



Consider equation (17)

$$\Delta O(\tau) = P_0 \tau_c \left[\frac{\tau}{\tau_c} \left(\frac{P_n}{P_0} - 1 \right) - \frac{P_n}{3P_0} \left(1 - e^{-\frac{3\tau}{\tau_c}} \right) \right]$$

Assume $\frac{\tau}{\tau_c} \geq 1$ (i.e. $e^{-\frac{3\tau}{\tau_c}} < 0.05$)

$$\therefore \Delta O(\tau) \approx P_0 \tau_c \left[\frac{\tau}{\tau_c} \left(\frac{P_n}{P_0} - 1 \right) - \frac{P_n}{3P_0} \right] \quad (28)$$

To solve for $\Delta O(\tau) = 0$...

$$0 = P_0 \tau_c \left[\frac{\tau}{\tau_c} \left(\frac{P_n}{P_0} - 1 \right) - \frac{P_n}{3P_0} \right]$$

$$\therefore \frac{\tau}{\tau_c} \left(\frac{P_n}{P_0} - 1 \right) = \frac{P_n}{3P_0} \quad (29)$$

$$\therefore \frac{\tau}{\tau_c} = \frac{\frac{P_n}{3P_0}}{\frac{P_n}{P_0} - 1} = \frac{1}{3} \cdot \frac{\frac{P_n}{P_0}}{\frac{P_n}{P_0} - 1} \quad (30)$$

$$= \frac{\frac{P_n}{3}}{P_n - P_0} = \frac{1}{3} \frac{P_n}{\frac{P_n}{P_0} - 1}$$

Productivity Increase $\frac{P_n}{P_0}$	Fixed Learning Curve $\frac{\tau}{\tau_c}$	Must hold $e^{-\frac{3\tau}{\tau_c}}$
1.05	7	
1.10	3.33	0.00004
1.15	2.5	0.00055
1.20	2	0.002
1.30	1.5	0.01
1.50	1	0.05